パワーエレクトロニクス
(舟木担当分)

第三回

•続続ダイオード整流回路
•倍電圧整流回路
•多相整流回路

平成17年6月13日月曜日　3限目

倍電圧整流回路

- 単相半波倍電圧整流回路
  - コッククロフト･ウォルトン回路とも言う
  - 絵
  - 動作
    - 脈動成分は電源周波数と同じ
  - もっと高い電圧も発生可能
    - 回路図
    - 動作

多相交流の整流回路

- 三相交流
  - 送電・動力電源で一般的に使用されている
- 三相半波整流回路
  - 抵抗負荷
    - 回路図
      - ΔY結線になっているのは，零相電流を流さないため
    - 三相交流電圧 $\begin{cases} v_{sa} = \sqrt{2}V \sin wt \\ v_{sb} = \sqrt{2}V \sin\left(wt - \frac{2}{3}p\right) \\ v_{sc} = \sqrt{2}V \sin\left(wt + \frac{2}{3}p\right) \end{cases}$
    - 波形の図
      - 電圧の最も高い相のダイオードがオン

三相半波整流回路

- 抵抗負荷
  - 直流平均電圧

$$\begin{aligned} E_d &= \frac{1}{2p}\int_0^{2p} e_d\, dwt \\ &= \frac{1}{2p}\left[\int_0^{\frac{p}{6}} v_{sc}\, dwt + \int_{\frac{p}{6}}^{\frac{5}{6}p} v_{sa}\, dwt + \int_{\frac{5p}{6}}^{\frac{3}{2}p} v_{sb}\, dwt + \int_{\frac{3}{2}p}^{2p} v_{sc}\, dwt\right] \\ &= \frac{3}{2p}\int_{\frac{p}{6}}^{\frac{5}{6}p} \sqrt{2}V \sin wt\, dwt \\ &= \frac{3V}{\sqrt{2}p}\left[-\cos wt\right]_{\frac{p}{6}}^{\frac{5}{6}p} = \frac{3V}{\sqrt{2}p}\left[\frac{\sqrt{3}}{2} + \frac{\sqrt{3}}{2}\right] = 3\sqrt{\frac{3}{2}}\frac{V}{p} \end{aligned}$$

$$\therefore E_d = 3\sqrt{\frac{3}{2}}\frac{V}{p}$$

三相全波整流回路

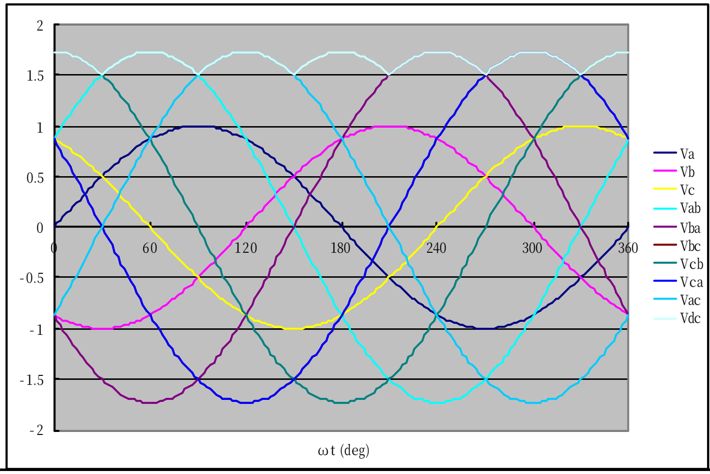
- 抵抗負荷
  - 回路図
    - グレエツ結線ともいう
    - 点弧の順番で番号付けしている
  - 三相交流電圧

$$\begin{cases} v_a = \sqrt{2}V \sin wt \\ v_b = \sqrt{2}V \sin\left(wt - \frac{2}{3}p\right) \\ v_c = \sqrt{2}V \sin\left(wt + \frac{2}{3}p\right) \end{cases} \quad \begin{cases} v_{cb} = v_c - v_b = \sqrt{6}V \cos wt \\ v_{ac} = v_a - v_c = \sqrt{6}V \cos\left(wt - \frac{2}{3}p\right) \\ v_{ba} = v_b - v_a = \sqrt{6}V \cos\left(wt + \frac{2}{3}p\right) \end{cases}$$

- 電圧の最も高くなる相の組み合わせでダイオードがオン
  - » 線間電圧の最も大きいものがオン

三相全波整流回路

- 抵抗負荷
  - 波形の図



三相全波整流回路

- 抵抗負荷
  - 直流平均電圧

$$E_d = \frac{1}{2p}\int_0^{2p} e_d\, dwt$$

$$= \frac{1}{2p}\left[\int_0^{\frac{p}{6}} v_c - v_b\, dwt + \int_{\frac{p}{6}}^{\frac{p}{2}} v_a - v_b\, dwt + \int_{\frac{p}{2}}^{\frac{5p}{6}} v_a - v_c\, dwt\right.$$

$$\left. + \int_{\frac{5p}{6}}^{\frac{7p}{6}} v_b - v_c\, dwt + \int_{\frac{7p}{6}}^{\frac{3p}{2}} v_b - v_a\, dwt + \int_{\frac{3p}{2}}^{\frac{11p}{6}} v_c - v_a\, dwt + \int_{\frac{11p}{6}}^{2p} v_c - v_b\, dwt\right]$$

$$= \frac{6}{2p}\int_{\frac{p}{6}}^{\frac{p}{2}}\left[\sqrt{2}V\sin wt - \sqrt{2}V\sin\left(wt - \tfrac{2}{3}p\right)\right]dwt$$

$$= \frac{3\sqrt{2}V}{p}\left[-\cos wt + \cos\left(wt - \tfrac{2}{3}p\right)\right]_{\frac{p}{6}}^{\frac{p}{2}} = \frac{3\sqrt{2}V}{p}\left[-0 + \tfrac{\sqrt{3}}{2} + \tfrac{\sqrt{3}}{2} - 0\right] = \frac{3\sqrt{6}V}{p}$$

$$\therefore E_d = \frac{3\sqrt{6}V}{p}$$

半波整流回路の2倍の直流電圧

三相全波整流回路

- 抵抗負荷
  - 高調波

$$E_{dk} = \frac{1}{2p}\int_0^{2p} e_d e^{-jkwt}\, dwt$$

$$= \frac{1}{2p}\left[\int_0^{\frac{p}{6}} v_{cb}e^{-jkwt}dwt + \int_{\frac{p}{6}}^{\frac{p}{2}} v_{ab}e^{-jkwt}dwt + \int_{\frac{p}{2}}^{\frac{5p}{6}} v_{ac}e^{-jkwt}dwt + \int_{\frac{5p}{6}}^{\frac{7p}{6}} v_{bc}e^{-jkwt}dwt + \int_{\frac{7p}{6}}^{\frac{3p}{2}} v_{ba}e^{-jkwt}dwt + \int_{\frac{3p}{2}}^{\frac{11p}{6}} v_{ca}e^{-jkwt}dwt + \int_{\frac{11p}{6}}^{2p} v_{cb}e^{-jkwt}dwt\right]$$

$$= \frac{1}{2p}\left[\int_{0,\frac{7p}{6},\frac{11p}{6}}^{\frac{p}{6},\frac{5p}{6},2p} v_{cb}e^{-jkwt}dwt + \int_{\frac{p}{2},\frac{7p}{6}}^{\frac{p}{6},\frac{3p}{2}} v_{ba}e^{-jkwt}dwt + \int_{\frac{11p}{6},\frac{p}{2}}^{\frac{3p}{2},\frac{5p}{6}} v_{ac}e^{-jkwt}dwt\right]$$

$$= \frac{\sqrt{6}V}{2p}\left[\int_{0,\frac{7p}{6},\frac{11p}{6}}^{\frac{p}{6},\frac{5p}{6},2p} \cos wt\, e^{-jkwt}dwt + \int_{\frac{p}{2},\frac{7p}{6}}^{\frac{p}{6},\frac{3p}{2}} \cos\left(wt + \tfrac{2}{3}p\right)e^{-jkwt}dwt + \int_{\frac{3p}{2},\frac{5p}{6}}^{\frac{11p}{6},\frac{p}{2}} \cos\left(wt - \tfrac{2}{3}p\right)e^{-jkwt}dwt\right]$$

$$= \frac{\sqrt{6}V}{4p}\left[\int_{0,\frac{7p}{6},\frac{11p}{6}}^{\frac{p}{6},\frac{5p}{6},2p}\left\{e^{jwt} + e^{-jwt}\right\}e^{-jkwt}dwt + \int_{\frac{p}{2},\frac{7p}{6}}^{\frac{p}{6},\frac{3p}{2}}\left\{e^{j\left(wt+\frac{2}{3}p\right)} + e^{-j\left(wt+\frac{2}{3}p\right)}\right\}e^{-jkwt}dwt + \int_{\frac{3p}{2},\frac{5p}{6}}^{\frac{11p}{6},\frac{p}{2}}\left\{e^{j\left(wt-\frac{2}{3}p\right)} + e^{-j\left(wt-\frac{2}{3}p\right)}\right\}e^{-jkwt}dwt\right]$$

$$= \frac{\sqrt{6}V}{4p}\left[\int_{0,\frac{7p}{6},\frac{11p}{6}}^{\frac{p}{6},\frac{5p}{6},2p}\left\{e^{j(1-k)wt} + e^{-j(1+k)wt}\right\}dwt + \int_{\frac{p}{2},\frac{7p}{6}}^{\frac{p}{6},\frac{3p}{2}}\left\{e^{j\left((1-k)wt+\frac{2}{3}p\right)} + e^{-j\left((1+k)wt+\frac{2}{3}p\right)}\right\}dwt + \int_{\frac{3p}{2},\frac{5p}{6}}^{\frac{11p}{6},\frac{p}{2}}\left\{e^{j\left((1-k)wt-\frac{2}{3}p\right)} + e^{-j\left((1+k)wt-\frac{2}{3}p\right)}\right\}dwt\right]$$

k=1

$$E_{d1} = \frac{\sqrt{6}V}{4p}\left[\int_{0,\frac{7p}{6},\frac{11p}{6}}^{\frac{p}{6},\frac{5p}{6},2p}\left\{1 + e^{-j2wt}\right\}dwt + \int_{\frac{p}{2},\frac{7p}{6}}^{\frac{p}{6},\frac{3p}{2}}\left\{1 + e^{-j\left(2wt+\frac{2}{3}p\right)}\right\}dwt + \int_{\frac{3p}{2},\frac{5p}{6}}^{\frac{11p}{6},\frac{p}{2}}\left\{1 + e^{-j\left(2wt-\frac{2}{3}p\right)}\right\}dwt\right]$$

$$= \frac{\sqrt{6}V}{4p}\left\{\left[wt + \frac{e^{-j2wt}}{-j2}\right]_{0,\frac{7p}{6},\frac{11p}{6}}^{\frac{p}{6},\frac{5p}{6},2p} + \left[wt + \frac{e^{-j\left(2wt+\frac{2}{3}p\right)}}{-j2}\right]_{\frac{p}{2},\frac{7p}{6}}^{\frac{p}{6},\frac{3p}{2}} + \left[wt + \frac{e^{-j\left(2wt-\frac{2}{3}p\right)}}{-j2}\right]_{\frac{3p}{2},\frac{5p}{6}}^{\frac{11p}{6},\frac{p}{2}}\right\}$$

$$= \frac{\sqrt{6}V}{4p}\left\{\left[wt\right]_{0,\frac{7p}{6},\frac{11p}{6},\frac{p}{2},\frac{7p}{6},\frac{3p}{2},\frac{5p}{6}}^{\frac{p}{6},\frac{5p}{6},2p,\frac{p}{6},\frac{3p}{2},\frac{11p}{6},\frac{p}{2}} + \frac{1}{-j2}\left[e^{-j2wt}\right]_{0,\frac{7p}{6},\frac{11p}{6},-\frac{p}{6},\frac{p}{2},\frac{13p}{6},\frac{3p}{2}}^{\frac{p}{6},\frac{5p}{6},2p,-\frac{p}{2},\frac{5p}{6},\frac{5p}{2},\frac{7p}{6}}\right\}$$

$$= \frac{\sqrt{6}V}{4p}\left\{\left[wt\right]_{0,\frac{7p}{6},\frac{7p}{6}}^{\frac{p}{6},2p,\frac{p}{6}} + \frac{1}{-j2}\left[e^{-j2wt}\right]_{\frac{11p}{6},-\frac{p}{6}}^{\frac{5p}{6},\frac{5p}{6}}\right\}$$

$$= \frac{\sqrt{6}V}{4p}\left\{\left[\frac{p}{6} + 2p + \frac{p}{6} - 0 - \frac{7p}{6} - \frac{7p}{6}\right] + \frac{1}{-j2}\left[e^{-j\frac{5p}{3}} + e^{-j\frac{5p}{3}} - e^{-j\frac{11p}{3}} - e^{j\frac{p}{3}}\right]\right\}$$

$$= \frac{\sqrt{6}V}{4p}\left\{\left[\frac{14p}{6} - \frac{14p}{6}\right] + \frac{1}{-j2}\left[e^{-j\frac{5p}{3}} + e^{-j\frac{5p}{3}} - e^{-j\frac{5p}{3}} - e^{-j\frac{5p}{3}}\right]\right\} = 0$$

三相全波整流回路

- 抵抗負荷
  - 高調波　k=-1

$$E_{d-1}=\frac{\sqrt{6}V}{4p}\left[\int_{0,\frac{7p}{6},\frac{11p}{6}}^{\frac{p}{6},\frac{5p}{6},2p}\left\{e^{j2wt}+1\right\}dwt+\int_{\frac{p}{2},\frac{7p}{6}}^{\frac{p}{6},\frac{3p}{2}}\left\{e^{j\left(2wt+\frac{2}{3}p\right)}+1\right\}dwt+\int_{\frac{3p}{2},\frac{5p}{6}}^{\frac{11p}{6},\frac{p}{2}}\left\{e^{j\left(2wt-\frac{2}{3}p\right)}+1\right\}dwt\right]$$

$$=\frac{\sqrt{6}V}{4p}\left\{\left[\frac{e^{j2wt}}{j2}+wt\right]_{0,\frac{7p}{6},\frac{11p}{6}}^{\frac{p}{6},\frac{5p}{6},2p}+\left[\frac{e^{j\left(2wt+\frac{2}{3}p\right)}}{j2}+wt\right]_{\frac{p}{2},\frac{7p}{6}}^{\frac{p}{6},\frac{3p}{2}}+\left[\frac{e^{j\left(2wt-\frac{2}{3}p\right)}}{j2}+wt\right]_{\frac{3p}{2},\frac{5p}{6}}^{\frac{11p}{6},\frac{p}{2}}\right\}$$

$$=\frac{\sqrt{6}V}{4p}\left\{\frac{1}{j2}\left[e^{j2wt}\right]_{0,\frac{7p}{6},\frac{11p}{6},\frac{7p}{6},\frac{11p}{6},\frac{5p}{6},\frac{p}{6}}^{\frac{p}{6},\frac{5p}{6},2p,\frac{5p}{6},\frac{13p}{6},\frac{7p}{6},-\frac{p}{6}}+\left[wt\right]_{0,\frac{7p}{6},\frac{11p}{6},\frac{p}{2},\frac{7p}{6},\frac{3p}{2},\frac{5p}{6}}^{\frac{p}{6},\frac{5p}{6},2p,\frac{p}{6},\frac{3p}{2},\frac{11p}{6},\frac{p}{2}}\right\}$$

$$=\frac{\sqrt{6}V}{4p}\left\{\frac{1}{j2}\left[e^{j2wt}\right]_{\frac{7p}{6},\frac{11p}{6},\frac{11p}{6}}^{\frac{5p}{6},\frac{13p}{6},-\frac{p}{6}}+\left[wt\right]_{0,\frac{7p}{6},\frac{7p}{6}}^{\frac{p}{6},2p,\frac{p}{6}}\right\}$$

$$=\frac{\sqrt{6}V}{4p}\left\{\frac{1}{j2}\left[e^{j\frac{5p}{3}}+e^{j\frac{13p}{3}}+e^{-j\frac{p}{3}}-e^{j\frac{7p}{3}}-e^{j\frac{11p}{3}}-e^{j\frac{11p}{3}}\right]+\left[\frac{p}{6}+2p+\frac{p}{6}-0-\frac{7p}{6}-\frac{7p}{6}\right]\right\}$$

$$=\frac{\sqrt{6}V}{4p}\left\{\frac{1}{j2}\left[e^{j\frac{5p}{3}}+e^{j\frac{p}{3}}+e^{j\frac{5p}{3}}-e^{j\frac{p}{3}}-e^{j\frac{5p}{3}}-e^{j\frac{5p}{3}}\right]+\left[\frac{14p}{6}-\frac{14p}{6}\right]\right\}=0$$

K≠±1

$$E_{dk}=\frac{\sqrt{6}V}{4p}\left\{\left[\frac{e^{j(1-k)wt}}{j(1-k)}+\frac{e^{-j(1+k)wt}}{-j(1+k)}\right]_{0,\frac{7p}{6},\frac{11p}{6}}^{\frac{p}{6},\frac{5p}{6},2p}+\left[\frac{e^{j\left((1-k)wt+\frac{2}{3}p\right)}}{j(1-k)}+\frac{e^{-j\left((1+k)wt+\frac{2}{3}p\right)}}{-j(1+k)}\right]_{\frac{p}{2},\frac{7p}{6}}^{\frac{p}{6},\frac{3p}{2}}+\left[\frac{e^{j\left((1-k)wt-\frac{2}{3}p\right)}}{j(1-k)}+\frac{e^{-j\left((1+k)wt-\frac{2}{3}p\right)}}{-j(1+k)}\right]_{\frac{3p}{2},\frac{5p}{6}}^{\frac{11p}{6},\frac{p}{2}}\right\}$$

$$=\frac{\sqrt{6}V}{4p}\left\{\left[\frac{e^{j(1-k)wt}}{j(1-k)}\right]_{0,\frac{7p}{6},\frac{11p}{6},\frac{7p}{6},\frac{11p}{6},\frac{5p}{6},\frac{p}{6}}^{\frac{p}{6},\frac{5p}{6},2p,\frac{5p}{6},\frac{13p}{6},\frac{7p}{6},-\frac{p}{6}}+\left[\frac{e^{-j(1+k)wt}}{-j(1+k)}\right]_{0,\frac{7p}{6},\frac{11p}{6},-\frac{p}{6},\frac{p}{2},\frac{13p}{6},\frac{3p}{2}}^{\frac{p}{6},\frac{5p}{6},2p,-\frac{p}{2},\frac{5p}{6},\frac{5p}{2},\frac{7p}{6}}\right\}$$

$$=\frac{\sqrt{6}V}{4p}\left\{\left[\frac{e^{j(1-k)wt}}{j(1-k)}\right]_{\frac{7p}{6},\frac{11p}{6}}^{\frac{5p}{6},\frac{13p}{6}}+\left[\frac{e^{-j(1+k)wt}}{-j(1+k)}\right]_{\frac{11p}{6},-\frac{p}{6}}^{\frac{5p}{6},\frac{5p}{6}}\right\}=\frac{\sqrt{6}V}{4p}\left\{\left[\frac{e^{j(1-k)wt}}{j(1-k)}\right]_{\frac{7p}{6},\frac{11p}{6}}^{\frac{5p}{6},\frac{p}{6}}+\left[\frac{e^{-j(1+k)wt}}{-j(1+k)}\right]_{\frac{11p}{6},\frac{11p}{6}}^{\frac{5p}{6},\frac{5p}{6}}\right\}$$

$$=\frac{\sqrt{6}V}{4p}\left\{\frac{e^{j(1-k)\frac{5p}{6}}+e^{j(1-k)\frac{13p}{6}}-e^{j(1-k)\frac{7p}{6}}-e^{j(1-k)\frac{11p}{6}}}{j(1-k)}+2\frac{e^{-j(1+k)\frac{5p}{6}}-e^{-j(1+k)\frac{11p}{6}}}{-j(1+k)}\right\}$$

$$=\frac{\sqrt{6}V}{4p}\left\{e^{j(1-k)\frac{5p}{6}}\frac{1+e^{j(1-k)\frac{4p}{3}}-e^{j(1-k)\frac{p}{3}}-e^{j(1-k)p}}{j(1-k)}+2e^{-j(1+k)\frac{5p}{6}}\frac{1-e^{-j(1+k)p}}{-j(1+k)}\right\}$$

$$=\frac{\sqrt{6}V}{4p}\left\{e^{j(1-k)\frac{5p}{6}}\frac{1-e^{j(1-k)\frac{p}{3}}-e^{j(1-k)p}\left[1-e^{j(1-k)\frac{p}{3}}\right]}{j(1-k)}+2e^{-j(1+k)\frac{5p}{6}}\frac{1-e^{-j(1+k)p}}{-j(1+k)}\right\}$$

$$=\frac{\sqrt{6}V}{4p}\left\{e^{j(1-k)\frac{5p}{6}}\frac{\left[1-e^{j(1-k)p}\right]\left[1-e^{j(1-k)\frac{p}{3}}\right]}{j(1-k)}+2e^{-j(1+k)\frac{5p}{6}}\frac{1-e^{-j(1+k)p}}{-j(1+k)}\right\}$$

➡ 偶数次数でE_{dk}=0

三相全波整流回路

- 誘導負荷
  - 電圧・電流の振る舞い
    - 電源電圧

$$\begin{cases} v_a=\sqrt{2}V\sin wt \\ v_b=\sqrt{2}V\sin\left(wt-\frac{2}{3}p\right) \\ v_c=\sqrt{2}V\sin\left(wt+\frac{2}{3}p\right)\end{cases}$$

    - 導通期間　360度
      - 各構成の導通角　60度

•回路図

•電圧波形

三相全波整流回路

- 誘導負荷
  - 電圧・電流の振る舞い
    - 周期定常状態を求めてみよう
      - ダイオードは順方向バイアス電圧印加で導通（点弧）
        後の講義で行うサイリスタの場合点弧制御可能）
        - » D1, D6導通時（π/6≦ωt ≦π/2）

$$e_d = L\tfrac{d}{dt}i_d + Ri_d = v_{ab}$$

        - » D1, D2導通時（π/2≦ωt ≦5π/6）

$$e_d = L\tfrac{d}{dt}i_d + Ri_d = -v_{ca}$$

        - » D2, D3導通時（5π/6≦ωt ≦7π/6）

$$e_d = L\tfrac{d}{dt}i_d + Ri_d = v_{bc}$$

        - » D3, D4導通時（π7/6≦ωt ≦3π/2）

$$e_d = L\tfrac{d}{dt}i_d + Ri_d = -v_{ab}$$

        - » D4, D5導通時（3π/2≦ωt ≦11π/6）

$$e_d = L\tfrac{d}{dt}i_d + Ri_d = v_{ca}$$

        - » D5, D6導通時（11π/6≦ωt ≦13π/6）

$$e_d = L\tfrac{d}{dt}i_d + Ri_d = -v_{bc}$$

三相全波整流回路

- 誘導負荷
  - 電圧・電流の振る舞い
    - 周期定常状態を求めてみよう
      - 解析条件
        - » D1,D4導通時, D3,D6導通時, D5,D2導通時各々同極性の直流を出力

$$e_d(wt) = e_d\left(wt+\tfrac{p}{3}\right) = e_d\left(wt+\tfrac{2p}{3}\right) = e_d(wt+p)$$
$$= e_d\left(wt+\tfrac{4p}{3}\right) = e_d\left(wt+\tfrac{5p}{3}\right) = e_d(wt+2p)$$

        - » D5→D1, D6→D2, D1→D3, D2→D4 , D3→D5 , D4→D6の転流時, 電流連続

$$i_d\left(\tfrac{p}{6}\right) = i_d\left(\tfrac{p}{2}\right) = i_d\left(\tfrac{5p}{6}\right) = i_d\left(\tfrac{7p}{6}\right) = i_d\left(\tfrac{3p}{2}\right) = i_d\left(\tfrac{11p}{6}\right)$$

      - ラプラス変換を用いて回路方程式を求解する
        - » 波形の対称性を利用して簡略化

三相全波整流回路

- 誘導負荷
  - 電圧 ·電流の振る舞い
    - 周期定常状態を求めてみよう (π/6≦ωt ≦π/2)

$$v_{ab} = \sqrt{6}V\sin\left(wt + \tfrac{p}{6}\right) = L\tfrac{d}{dt}i_d + Ri_d$$

– 時間の原点をπ/6ずらす

$$\sqrt{6}V\sin\left(wt + \tfrac{p}{3}\right) = L\tfrac{d}{dt}i_d + Ri_d$$

$$\sin\left(wt + \tfrac{p}{3}\right) = \sin wt\cos\tfrac{p}{3} + \cos wt\sin\tfrac{p}{3} = \tfrac{1}{2}\sin wt + \tfrac{\sqrt{3}}{2}\cos wt$$

$$\sqrt{6}V\left(\tfrac{1}{2}\tfrac{w}{s^2+w^2} + \tfrac{\sqrt{3}}{2}\tfrac{s}{s^2+w^2}\right) = LsI_d - Li_{d0} + RI_d \qquad i_{dt=0} \neq 0$$

$$I_d(Ls+R) = \tfrac{\sqrt{6}}{2}V\left(\tfrac{w}{s^2+w^2} + \sqrt{3}\tfrac{s}{s^2+w^2}\right) + Li_{d0}$$

$$I_d = \tfrac{\sqrt{6}}{2}V\tfrac{1}{Ls+R}\left(\tfrac{w}{s^2+w^2} + \sqrt{3}\tfrac{s}{s^2+w^2}\right) + Li_{d0}\tfrac{1}{Ls+R}$$

三相全波整流回路

- 誘導負荷
  - 電圧 ·電流の振る舞い
    - 周期定常状態を求めてみよう (π/6≦ωt ≦π/2)

$$\tfrac{1}{Ls+R}\left(\tfrac{w}{s^2+w^2} + \sqrt{3}\tfrac{s}{s^2+w^2}\right) = \tfrac{\sqrt{3}s+w}{(Ls+R)(s^2+w^2)} = \tfrac{a}{Ls+R} + \tfrac{bs+c}{s^2+w^2} = \tfrac{(a+bL)s^2+(bR+cL)s+aw^2+cR}{(Ls+R)(s^2+w^2)}$$

$$\begin{cases} a+bL=0 \\ bR+cL=\sqrt{3} \\ aw^2+cR=w \end{cases} \qquad \begin{cases} a=\frac{-\sqrt{3}RL+wL^2}{R^2+w^2L^2} \\ b=\frac{\sqrt{3}R-wL}{R^2+w^2L^2} \\ c=\frac{wR+\sqrt{3}w^2L}{R^2+w^2L^2} \end{cases}$$

$$\begin{aligned} I_d &= \tfrac{\sqrt{6}}{2}V\left(\tfrac{a}{Ls+R} + \tfrac{bs+c}{s^2+w^2}\right) + Li_{d0}\tfrac{1}{Ls+R} \\ &= \tfrac{\sqrt{6}}{2}V\left(\tfrac{-\sqrt{3}RL+wL^2}{R^2+w^2L^2}\tfrac{1}{Ls+R} + \tfrac{\sqrt{3}R-wL}{R^2+w^2L^2}\tfrac{s}{s^2+w^2} + \tfrac{wR+\sqrt{3}w^2L}{R^2+w^2L^2}\tfrac{1}{s^2+w^2}\right) + Li_{d0}\tfrac{1}{Ls+R} \\ &= \tfrac{\sqrt{6}}{2}V\left(\tfrac{-\sqrt{3}R+wL}{R^2+w^2L^2}\tfrac{1}{s+\frac{R}{L}} + \tfrac{\sqrt{3}R-wL}{R^2+w^2L^2}\tfrac{s}{s^2+w^2} + \tfrac{R+\sqrt{3}L}{R^2+w^2L^2}\tfrac{w}{s^2+w^2}\right) + i_{d0}\tfrac{1}{s+\frac{R}{L}} \\ &= \tfrac{\sqrt{6}}{2}V\tfrac{1}{R^2+w^2L^2}\left[\left(-\sqrt{3}R+wL\right)\tfrac{1}{s+\frac{R}{L}} + \left(\sqrt{3}R-wL\right)\tfrac{s}{s^2+w^2} + \left(R+\sqrt{3}wL\right)\tfrac{w}{s^2+w^2}\right] + i_{d0}\tfrac{1}{s+\frac{R}{L}} \end{aligned}$$

三相全波整流回路

- 誘導負荷
  - 電圧・電流の振る舞い
    - 周期定常状態を求めてみよう

$$i_d(t)=\frac{\sqrt{6}}{2}V\frac{1}{R^2+w^2L^2}\left[\left(-\sqrt{3}R+wL\right)\exp\left(-\frac{R}{L}t\right)+\left(\sqrt{3}R-wL\right)\cos wt+\left(\sqrt{3}wL+R\right)\sin wt\right]$$
$$+i_{d0}\exp\left(-\frac{R}{L}t\right)$$

  - 半波対称・電流連続条件

$$\begin{aligned}i_d(0)&=i_{d0}=i_d\left(\frac{p}{3w}\right)\\&=\frac{\sqrt{6}}{2}V\frac{1}{R^2+w^2L^2}\left[\left(-\sqrt{3}R+wL\right)\exp\left(-\frac{R}{L}\frac{p}{3w}\right)+\left(\sqrt{3}R-wL\right)\cos\frac{p}{3}+\left(\sqrt{3}wL+R\right)\sin\frac{p}{3}\right]\\&\quad+i_{d0}\exp\left(-\frac{R}{L}\frac{p}{3w}\right)\\&=\frac{\sqrt{6}}{2}V\frac{1}{R^2+w^2L^2}\left[\left(-\sqrt{3}R+wL\right)\exp\left(-\frac{R}{L}\frac{p}{3w}\right)+\left(\sqrt{3}R-wL\right)\frac{1}{2}+\left(\sqrt{3}wL+R\right)\frac{\sqrt{3}}{2}\right]\\&\quad+i_{d0}\exp\left(-\frac{R}{L}\frac{p}{3w}\right)\end{aligned}$$

三相全波整流回路

- 誘導負荷
  - 電圧・電流の振る舞い
    - 周期定常状態を求めてみよう

$$i_{d0}=\frac{\sqrt{6}}{2}V\frac{1}{R^2+w^2L^2}\left[\left(-\sqrt{3}R+wL\right)\exp\left(-\frac{R}{L}\frac{p}{3w}\right)+\sqrt{3}R+wL\right]+i_{d0}\exp\left(-\frac{R}{L}\frac{p}{3w}\right)$$

$$i_{d0}\left[1-\exp\left(-\frac{R}{L}\frac{p}{3w}\right)\right]=\frac{\sqrt{6}}{2}V\frac{1}{R^2+w^2L^2}\left[\left(-\sqrt{3}R+wL\right)\exp\left(-\frac{R}{L}\frac{p}{3w}\right)+\sqrt{3}R+wL\right]$$

$$i_{d0}=\frac{\sqrt{6}}{2}V\frac{1}{R^2+w^2L^2}\frac{\left(-\sqrt{3}R+wL\right)\exp\left(-\frac{R}{L}\frac{p}{3w}\right)+\sqrt{3}R+wL}{1-\exp\left(-\frac{R}{L}\frac{p}{3w}\right)}$$

$$i_d(t)=\frac{\sqrt{6}}{2}V\frac{1}{R^2+w^2L^2}\left[\left(-\sqrt{3}R+wL\right)\exp\left(-\frac{R}{L}t\right)+\left(\sqrt{3}R-wL\right)\cos wt+\left(\sqrt{3}wL+R\right)\sin wt\right]$$
$$+\frac{\sqrt{6}}{2}V\frac{1}{R^2+w^2L^2}\frac{\left(-\sqrt{3}R+wL\right)\exp\left(-\frac{R}{L}\frac{p}{3w}\right)+\sqrt{3}R+wL}{1-\exp\left(-\frac{R}{L}\frac{p}{3w}\right)}\exp\left(-\frac{R}{L}t\right)$$

三相全波整流回路

- 誘導負荷
  - 電圧・電流の振る舞い
    - 周期定常状態を求めてみよう

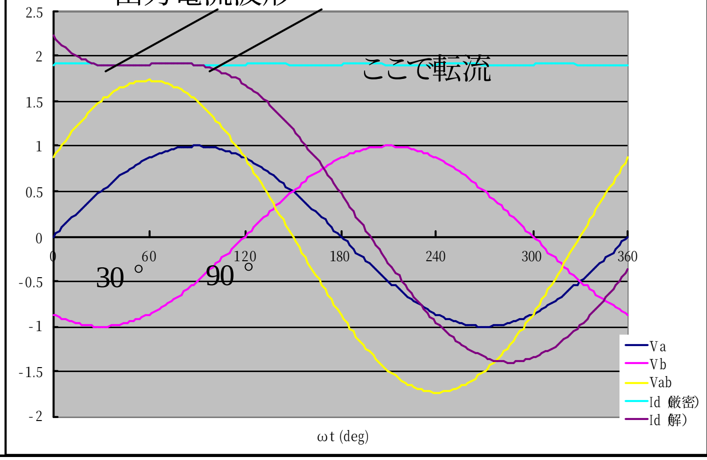
$$i_d(t)=\frac{\sqrt{6}}{2}V\frac{1}{R^2+w^2L^2}\left[\frac{2wL}{1-\exp\left(-\frac{R}{L}\frac{p}{3w}\right)}\exp\left(-\frac{R}{L}t\right)+\left(\sqrt{3}R-wL\right)\cos wt+\left(\sqrt{3}wL+R\right)\sin wt\right]$$

    - 時間の原点を元に戻す

$$i_d(t)=\frac{\sqrt{6}}{2}V\frac{1}{R^2+w^2L^2}$$
$$\left[\frac{2wL}{1-\exp\left(-\frac{R}{L}\frac{p}{3w}\right)}\exp\left(-\frac{R}{wL}\left\{wt-\frac{p}{6}\right\}\right)+\left(\sqrt{3}R-wL\right)\cos\left\{wt-\frac{p}{6}\right\}+\left(\sqrt{3}wL+R\right)\sin\left\{wt-\frac{p}{6}\right\}\right]$$

三相全波整流回路

- 誘導負荷
  - 電圧・電流の振る舞い
    - 出力電流波形



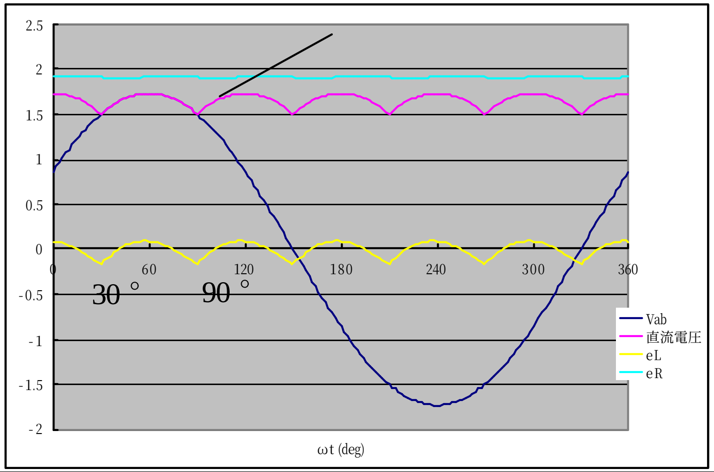
3相全波整流回路

- 誘導負荷
  - 各部の電圧波形を求める
    - 負荷抵抗電圧
      - 電流と相似波形　$e_R = Ri_d$
    - インダクタ電圧

$$e_L = L\frac{d}{dt}i_d = \frac{\sqrt{6}}{2}V\frac{L}{R^2+w^2L^2}$$
$$\left[\frac{-2wR}{1-\exp\left(-\frac{R}{L}\frac{p}{3w}\right)}\exp\left(-\frac{R}{wL}\left\{wt-\frac{p}{6}\right\}\right)\right.$$
$$\left.-\left(\sqrt{3}R-wL\right)w\sin\left\{wt-\frac{p}{6}\right\}+\left(\sqrt{3}wL+R\right)w\cos\left\{wt-\frac{p}{6}\right\}\right]$$

3相全波整流回路

- 誘導負荷
  - 各部の電圧波形を求める



三相全波整流回路

- 誘導負荷
  - 直流出力電圧平均値
    - インダクタ電圧
      - 定常状態では，エネルギーの入出力が均衡

$$
\begin{aligned}
E_{dL} &= \frac{3}{p}\int_{\frac{p}{6}}^{\frac{p}{2}} e_L \, dwt \\
&= \frac{3}{p}\frac{\sqrt{6}}{2}V \frac{1}{R^2+w^2L^2} \\
&\quad \left[\frac{2wL}{1-\exp\left(-\frac{R}{L}\frac{p}{3w}\right)}\exp\left(-\frac{R}{wL}\left\{wt-\frac{p}{6}\right\}\right)+\left(\sqrt{3}R-wL\right)\cos\left\{wt-\frac{p}{6}\right\}+\left(\sqrt{3}wL+R\right)\sin\left\{wt-\frac{p}{6}\right\}\right]_{\frac{p}{6}}^{\frac{p}{2}} \\
&= \frac{3}{p}\frac{\sqrt{6}}{2}V \frac{1}{R^2+w^2L^2} \\
&\quad \left[\frac{2wL}{1-\exp\left(-\frac{R}{L}\frac{p}{3w}\right)}\left\{\exp\left(-\frac{R}{wL}\frac{p}{3}\right)-1\right\}+\left(\sqrt{3}R-wL\right)\left\{\cos\frac{p}{3}-1\right\}+\left(\sqrt{3}wL+R\right)\left\{\sin\frac{p}{3}-0\right\}\right] \\
&= \frac{3}{p}\frac{\sqrt{6}}{2}V \frac{1}{R^2+w^2L^2}\left[-2wL-\frac{\sqrt{3}R-wL}{2}+\sqrt{3}\frac{\sqrt{3}wL+R}{2}\right] \\
&= 0
\end{aligned}
$$

三相全波整流回路

- 誘導負荷
  - 直流出力電圧平均値
    - 負荷抵抗電圧
      - 電流と相似波形

$$
\begin{aligned}
E_{dR} &= \frac{3}{p}\int_{\frac{p}{6}}^{\frac{p}{2}} e_R \, dwt = \frac{3}{p}\int_{\frac{p}{6}}^{\frac{p}{2}} R i_d \, dwt \\
&= \frac{3}{p}\frac{\sqrt{6}}{2}V \frac{R}{R^2+w^2L^2} \\
&\quad \left[\frac{2wL}{1-\exp\left(-\frac{R}{L}\frac{p}{3w}\right)}\frac{-wL}{R}\exp\left(-\frac{R}{wL}\left\{wt-\frac{p}{6}\right\}\right)+\left(\sqrt{3}R-wL\right)\sin\left\{wt-\frac{p}{6}\right\}-\left(\sqrt{3}wL+R\right)\cos\left\{wt-\frac{p}{6}\right\}\right]_{\frac{p}{6}}^{\frac{p}{2}} \\
&= \frac{3}{p}\frac{\sqrt{6}}{2}V \frac{R}{R^2+w^2L^2} \\
&\quad \left[\frac{2wL}{1-\exp\left(-\frac{R}{L}\frac{p}{3w}\right)}\frac{-wL}{R}\left\{\exp\left(-\frac{R}{wL}\frac{p}{3}\right)-1\right\}+\left(\sqrt{3}R-wL\right)\left\{\sin\frac{p}{3}-0\right\}-\left(\sqrt{3}wL+R\right)\left\{\cos\frac{p}{3}-1\right\}\right] \\
&= \frac{3}{p}\frac{\sqrt{6}}{2}V \frac{R}{R^2+w^2L^2}\left[\frac{2w^2L^2}{R}+\sqrt{3}\frac{\sqrt{3}R-wL}{2}+\frac{\sqrt{3}wL+R}{2}\right] = \frac{3}{p}\frac{\sqrt{6}}{2}V\frac{R}{R^2+w^2L^2}\left[\frac{2w^2L^2}{R}+2R\right] \\
&= \frac{3\sqrt{6}}{p}V
\end{aligned}
$$

抵抗負荷時と同じ

三相全波整流回路

- 容量負荷
  - 回路図
  - 回路の動作
    - » D1, D6導通，全てオフ　(π/6≦ωt ≦π/2)
    - » D1, D2導通，全てオフ　(π/2≦ωt ≦5π/6)
    - » D2, D3導通，全てオフ　(5π/6≦ ωt ≦7π/6)
    - » D3, D4導通，全てオフ　(π 7/6≦ ωt ≦3π/2)
    - » D4, D5導通，全てオフ　(3π/2≦ ωt ≦11π/6)
    - » D5, D6導通，全てオフ　(11π/6≦ ωt ≦13π/6)
  - 導通の状態が一周期に6回
    - 単相整流回路容量性負荷同様に解析してみよう

•回路図

•電圧波形

三相全波整流回路

- 容量負荷
  - 電圧・電流の振る舞い
    - 電源電圧
      - 三相のうち二相

$$\begin{cases} v_a = \sqrt{2}V \sin wt \\ v_b = \sqrt{2}V \sin\left(wt - \frac{2}{3}p\right) \\ v_c = \sqrt{2}V \sin\left(wt + \frac{2}{3}p\right) \end{cases}$$

- 導通期間中，負荷電圧は電源電圧と等しい
  - オンは，電源電圧と負荷電圧が等しくなった時点
  - オン時，Cを充電するため大電流が流れる（可能性）

$$e_d = \pm \begin{cases} v_a - v_b \\ v_b - v_c \\ v_c - v_a \end{cases} \qquad i_d = i_C + i_R = C\frac{d}{dt}e_d + \frac{e_d}{R}$$

- 非導通期間中，RCで閉回路を構成
  - Rを介してCが放電

$$i_C = -i_R \qquad i_R = -C\frac{d}{dt}e_d$$

三相全波整流回路

- 容量負荷
  - 出力波形を求める（π/6≦ωt ≦π/2分）
    - Vabが印加される　$v_{ab} = \sqrt{6}V\sin\left(wt + \frac{p}{6}\right)$
    - 導通開始点　θ $_{on}$
      - コンデンサ電圧初期値をv_{c0}とする

$$v = v_{C0} = \sqrt{6}V\sin\left(q_{on} + \frac{p}{6}\right)$$

    - 導通終了点　θ $_{off}$
      - i_dが0となる　$e_d = v$
      - 導通期間中

$$i_d\left(wt = q_{off}\right) = C\frac{d}{dt}e_d + \frac{e_d}{R} = 0$$

$\frac{p}{3} < q_{off}$　になるので

$$C\sqrt{6}Vw\cos\left(q_{off} + \frac{p}{6}\right) + \frac{\sqrt{6}V\sin\left(q_{off} + \frac{p}{6}\right)}{R} = 0 \qquad \therefore q_{off} = \frac{5}{6}p - \arctan RwC$$

三相全波整流回路

- 容量負荷
  - 出力波形を求める
    - 非導通期間中

$$i_R = -C\frac{d}{dt}e_d \qquad \frac{e_d}{R} = -C\frac{d}{dt}e_d$$

$$\frac{E_d}{R} = -C\left(sE_d - e_{d0}\right)$$

但し$e_{d0} = \sqrt{6}V\sin\left(q_{off} + \frac{p}{6}\right)$

$$E_d = \frac{e_{d0}}{s + \frac{1}{RC}}$$

    - 非導通開始点θ $_{off}$
      - 出力電圧　$e_d = e_{d0}e^{-\frac{1}{RwC}\left(wt - q_{off}\right)}$

•波形の図

三相全波整流回路

- 容量負荷
  - 出力波形を求める
    - v_{c0}とe_{d0}の接続条件（非導通→導通時点） 単相との相違

$$e_d\left(\omega t = \tfrac{\pi}{3} + \theta_{on}\right) = e_{d0}\, e^{-\frac{1}{R\omega C}\left(\frac{\pi}{3}+\theta_{on}-\theta_{off}\right)} = v_{c0}$$

$$v_{c0} = e_{d0}\, e^{-\frac{1}{R\omega C}\left(\frac{\pi}{3}+\theta_{on}-\theta_{off}\right)}$$

$$v_{C0} = \sqrt{6}V\sin\left(\theta_{on}+\tfrac{\pi}{6}\right) \qquad e_{d0} = \sqrt{6}V\sin\left(\theta_{off}+\tfrac{\pi}{6}\right)$$

より

$$\sqrt{6}V\sin\left(\theta_{on}+\tfrac{\pi}{6}\right) = \sqrt{2}V\sin\left(\theta_{off}+\tfrac{\pi}{6}\right)e^{-\frac{1}{R\omega C}\left(\frac{\pi}{3}+\theta_{on}-\theta_{off}\right)}$$

$$\sin\left(\theta_{on}+\tfrac{\pi}{6}\right) = \sin\left(\theta_{off}+\tfrac{\pi}{6}\right)e^{-\frac{1}{R\omega C}\left(\frac{\pi}{3}+\theta_{on}-\theta_{off}\right)}$$

の解としてθ_{on}が求まる

三相全波整流回路

- 容量負荷
  - リップル（脈動）成分を求める
    - 直流出力電圧最大値 $e_{d-\max} = \sqrt{6}V$
    - 直流出力電圧最小値 $e_{d-\min} = \sqrt{6}V\sin\left(\theta_{on}+\tfrac{\pi}{6}\right)$
    - 電圧脈動幅 $\Delta V = e_{d-\max} - e_{d-\min}$

$$= \sqrt{6}V\left[1-\sin\left(\theta_{on}+\tfrac{\pi}{6}\right)\right]$$

    - 仮定により簡略化し，脈動幅を求める
      - 仮定 平滑コンデンサ容量が十分大きい $\theta_{off} \cong \frac{\pi}{3}$

ちょっと苦しいけど $\theta_{on} \cong \frac{\pi}{3}$

$$\sin\left(\theta_{on}+\tfrac{\pi}{6}\right) = \sin\left(\theta_{off}+\tfrac{\pi}{6}\right)e^{-\frac{1}{R\omega C}\left(\frac{\pi}{3}+\theta_{on}-\theta_{off}\right)} = \sin\tfrac{\pi}{2}\, e^{-\frac{1}{R\omega C}\left(\frac{\pi}{3}+\frac{\pi}{3}-\frac{\pi}{3}\right)} = e^{-\frac{\pi}{3R\omega C}}$$

  - 電圧脈動幅 $\Delta V = \sqrt{6}V\left(1-\sin\theta_{on}\right) = \sqrt{6}V\left(1-e^{-\frac{\pi}{3R\omega C}}\right)$
    - » RC大で脈動が小さくなる

$$e^{-\frac{\pi}{3R\omega C}} \cong 1-\frac{\pi}{3R\omega C} \;\Rightarrow\; \Delta V \cong \sqrt{6}V\frac{\pi}{3R\omega C}$$

単相の
3分の1

三相全波整流回路

- 容量負荷
  - ダイオード電流ピーク値を求める
    - 整流回路は容量負荷にパルス状の電流を流す
      - ダイオード電流

$$i_d = i_R + i_c = \frac{e_d}{R} + C\frac{d}{dt}e_d = \tfrac{1}{R}\sqrt{6}V\sin\left(wt+\tfrac{p}{6}\right) + Cw\sqrt{6}V\cos\left(wt+\tfrac{p}{6}\right)$$

» 導通期間が小さい場合
ωT=π/3付近 $\quad i_d \cong \tfrac{1}{R}\sqrt{6}V + Cw\sqrt{6}V\cos\left(wt+\tfrac{p}{6}\right)$

– ダイオードの導通期間Δtを求める

$$\sqrt{6}V\sin\left(\tfrac{p}{3} - w\Delta t + \tfrac{p}{6}\right) = \sqrt{6}V\cos w\Delta t = \sqrt{6}V - \Delta V \cong \sqrt{6}V\left(1-\tfrac{p}{3RwC}\right)$$

$$\cos w\Delta t \cong 1 - \tfrac{p}{3RwC}$$

$$\cos x \cong 1 - \tfrac{1}{2}x^2 \;\Rightarrow\; 1 - \frac{1}{2}(w\Delta t)^2 \cong 1 - \frac{p}{3RwC} \;\Rightarrow\; \Delta t \cong \sqrt{\tfrac{2p}{3Rw^3C}}$$

三相全波整流回路

- 容量負荷
  - ダイオード電流を求める
    - ダイオード平均電流 i_{c-ave}
      - 平滑コンデンサへの充電電荷

$$Q_{charge} = C\Delta V = i_{c-ave}\Delta t$$

$$i_{c-ave} = C\tfrac{\Delta V}{\Delta t} = C\frac{\sqrt{6}V\frac{p}{3RwC}}{\sqrt{\frac{2p}{3Rw^3C}}} = V\sqrt{\frac{pw}{RC}}$$

– ダイオードピーク電流

$$\begin{aligned} i_{d-\max} &= i_d\left(\tfrac{p}{3} - \Delta t\right) = Cw\sqrt{6}V\cos w\Delta t + \tfrac{\sqrt{6}V}{R} \\ &= Cw\sqrt{6}V\left(1 - \tfrac{p}{3RwC}\right) + \tfrac{\sqrt{6}V}{R} \\ &= \sqrt{6}V\left(wC - \tfrac{p}{3R} + \tfrac{1}{R}\right) \end{aligned}$$

- 単相より小さくなる

位相制御よる交直変換器の出力調整

- ダイオード
  - ON・OFF共非可制御
    - 交直変換は整流のみ可能
- サイリスタ
  - ONはゲート信号で制御可能
    - 但し, 順バイアス印加時のみ
  - OFFは非可制御
    - 但し, ON時の状態がゲート信号で変わるため, OFF時の状態も付随して変化する
  - 回路構成・条件によっては整流・逆変換の双方向変換が可能

位相制御単相半波整流回路

- 抵抗負荷
  - 直流出力電圧平均値
    - 導通期間 α～π 点弧角α）
      - ダイオードでは0～π
      - 電源電圧 $v = \sqrt{2}V \sin wt$

回路の絵
波形の絵

$$E_d = \frac{1}{2p}\int_0^{2p} e_d dwt = \frac{1}{2p}\int_a^p vdwt = \frac{1}{2p}\int_a^p \sqrt{2}V \sin wtdwt$$

$$= \frac{V}{\sqrt{2}p}\left[-\cos wt\right]_a^p = \frac{V}{\sqrt{2}p}\left[1+\cos a\right] = \frac{\sqrt{2}V}{p}\frac{1+\cos a}{2}$$

$$= \frac{\sqrt{2}V}{p}\frac{\cos^2\frac{a}{2}+\sin^2\frac{a}{2}+\cos^2\frac{a}{2}-\sin^2\frac{a}{2}}{2} = \frac{\sqrt{2}V}{p}\cos^2\frac{a}{2}$$

ダイオード
はα=0に
相当