応用システム工学
第五回　回帰分析

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重回帰分析

分散

- 分散

$$s_{xx} = \frac{1}{n}\sum_{i=1}^{n}\left(x_i - \overline{x}\right)^2 = \frac{1}{n}\sum_{i=1}^{n} {x_i}^2 - \overline{x}^2$$

- 共分散

$$s_{xy} = \frac{1}{n}\sum_{i=1}^{n}\left(x_i - \overline{x}\right)\left(y_i - \overline{y}\right) = \frac{1}{n}\sum_{i=1}^{n} x_i y_i - \overline{xy}$$

回帰直線の予測誤差

- yのxへの回帰直線によるyの予測値Y

$$Y = \frac{s_{xy}}{s_{xx}}\left(x - \overline{x}\right) + \overline{y}$$

  – 予測誤差 $e_i = y_i - Y_i$

  – 予測誤差の標準偏差

$$s_e = \sqrt{\frac{1}{n-2}\sum_{i=1}^{n}\left(e_i - \overline{e}\right)^2} = \sqrt{\frac{1}{n-2}\sum_{i=1}^{n}{e_i}^2}$$

    - 回帰式の $\hat{a}_0, \hat{a}_1$ は，目的変数を用いて求めているため，これを差し引いたn-2で割る。
    - 回帰式がk個の定数を持つ場合n-kで割る

回帰直線の予測誤差

n個のデータの
目的変数yと
説明変数xの組

	目的変数:y	説明変数:x	予測誤差
1	y_1	x_1	$e_1=y_1-(a_0+a_1x_1)$
2	y_2	x_2	$e_2=y_2-(a_0+a_1x_2)$
…			
i	y_i	x_i	$e_i=y_i-(a_0+a_1x_i)$
…			
n	y_n	x_n	$e_n=y_n-(a_0+a_1x_n)$

予測誤差の平均

$$\bar{e} = \frac{1}{n}\sum_{i=1}^{n} e_i = \frac{1}{n}\sum_{i=1}^{n}\left(y_i - Y_i\right)$$

$$= \frac{1}{n}\sum_{i=1}^{n}\left\{ y_i - \left[\frac{s_{xy}}{s_{xx}}\left(x_i - \bar{x}\right) + \bar{y} \right] \right\} = \frac{1}{n}\sum_{i=1}^{n}\left\{ y_i - \bar{y} + \frac{s_{xy}}{s_{xx}}\left(x_i - \bar{x}\right) \right\}$$

$$= \frac{1}{n}\sum_{i=1}^{n} y_i - \frac{1}{n}\sum_{i=1}^{n}\bar{y} + \frac{s_{xy}}{s_{xx}}\left(\frac{1}{n}\sum_{i=1}^{n} x_i - \frac{1}{n}\sum_{i=1}^{n}\bar{x} \right)$$

$$= \bar{y} - \frac{1}{n}\bar{y}n + \frac{s_{xy}}{s_{xx}}\left(\bar{x} - \frac{1}{n}\bar{x}n \right) = \bar{y} - \bar{y} + \frac{s_{xy}}{s_{xx}}\left(\bar{x} - \bar{x}\right) = 0$$

相関係数

- 変量間の関係の強さを表す
  - 変量x,y間の相関係数 $r_{xy} = \dfrac{s_{xy}}{\sqrt{s_{xx}s_{yy}}}$
    - 決定係数 $r_{xy}{}^{2}$
- 予測誤差の標準偏差と相関係数の関係

$$s_e{}^2 = \frac{1}{n-2}\sum_{i=1}^{n} e_i{}^2 = \frac{n}{n-2}\frac{1}{n}\sum_{i=1}^{n}\left(y_i - Y_i\right)^2$$

$$= \frac{n}{n-2}\frac{1}{n}\sum_{i=1}^{n}\left\{y_i - \left[\frac{s_{xy}}{s_{xx}}\left(x_i - \bar{x}\right) + \bar{y}\right]\right\}^2$$

相関係数

$$s_e{}^2 = \frac{n}{n-2}\frac{1}{n}\sum_{i=1}^{n}\left\{y_i - \bar{y} - \frac{s_{xy}}{s_{xx}}(x_i - \bar{x})\right\}^2$$

$$= \frac{n}{n-2}\frac{1}{n}\left\{\sum_{i=1}^{n}(y_i - \bar{y})^2 - 2\frac{s_{xy}}{s_{xx}}\sum_{i=1}^{n}(y_i - \bar{y})(x_i - \bar{x}) + \left(\frac{s_{xy}}{s_{xx}}\right)^2\sum_{i=1}^{n}(x_i - \bar{x})^2\right\}$$

$$= \frac{n}{n-2}\left\{s_{yy} - 2\frac{s_{xy}}{s_{xx}}s_{xy} + \left(\frac{s_{xy}}{s_{xx}}\right)^2 s_{xx}\right\} = \frac{n}{n-2}\left\{s_{yy} - \frac{s_{xy}{}^2}{s_{xx}}\right\}$$

$$= \frac{n}{n-2}s_{yy}\left\{1 - \frac{s_{xy}{}^2}{s_{xx}s_{yy}}\right\} = \frac{n}{n-2}s_{yy}\left\{1 - r_{xy}{}^2\right\}$$

相関係数

$$s_e{}^2 = \frac{n}{n-2} s_{yy} \left\{ 1 - r_{xy}{}^2 \right\}$$

– 相関係数の値の範囲

$\frac{n}{n-2} s_{yy} \left\{ 1 - r_{xy}{}^2 \right\} = s_e{}^2 \geq 0$ より $s_{yy} \geq 0$ だから

$$1 - r_{xy}{}^2 \geq 0$$

$$-1 \leq r_{xy} \leq 1$$

– 相関係数が $r_{xy} = \pm 1$ のとき，直線上にある

$$s_e = 0$$

– 相関係数が0に近いほど，x,yの線形的な関係は小さくなる

決定係数

- 相関係数の方が予測誤差の評価に適当
  - 相関係数 r_{xy} は，一次変換（単位変換等）
    $X = ax + b \qquad Y = cy + d$
    によって変化しない
  - 予測誤差の標準偏差 s_e は，一次変換により変化する
- 決定係数（寄与率）→相関係数の２乗

$$r_{xy}{}^2 = \frac{s_{xy}{}^2}{s_{xx}s_{yy}} = 1 - \frac{\sum_{i=1}^{n}(y_i - Y_i)^2}{\sum_{i=1}^{n}(y_i - \bar{y})^2}$$

とも書ける⇒確認

決定係数

$$\sum_{i=1}^{n}(y_i - Y_i)^2 = \sum_{i=1}^{n}\left\{y_i - \left[\frac{s_{xy}}{s_{xx}}(x_i - \bar{x}) + \bar{y}\right]\right\}^2$$

$$= \sum_{i=1}^{n}\left\{y_i - \bar{y} - \frac{s_{xy}}{s_{xx}}(x_i - \bar{x})\right\}^2$$

$$= \sum_{i=1}^{n}(y_i - \bar{y})^2 - 2\frac{s_{xy}}{s_{xx}}\sum_{i=1}^{n}(y_i - \bar{y})(x_i - \bar{x}) + \left(\frac{s_{xy}}{s_{xx}}\right)^2\sum_{i=1}^{n}(x_i - \bar{x})^2$$

$$= ns_{yy} - 2\frac{s_{xy}}{s_{xx}}ns_{xy} + \left(\frac{s_{xy}}{s_{xx}}\right)^2 ns_{xx} = n\left(s_{yy} - \frac{{s_{xy}}^2}{s_{xx}}\right)$$

決定係数

$$1-\frac{\sum_{i=1}^{n}\left(y_i-Y_i\right)^2}{\sum_{i=1}^{n}\left(y_i-\bar{y}\right)^2}=1-\frac{n\left(s_{yy}-\frac{{s_{xy}}^2}{s_{xx}}\right)}{ns_{yy}}$$

$$=1-\frac{s_{yy}-\frac{{s_{xy}}^2}{s_{xx}}}{s_{yy}}$$

$$=1-\left(1-\frac{{s_{xy}}^2}{s_{xx}s_{yy}}\right)=\frac{{s_{xy}}^2}{s_{xx}s_{yy}}={r_{xy}}^2$$

変数変換による線形回帰

- 変量x,yの関係は必ずしも線形ではない
  - 線形回帰モデル

$$y_i = a_0 + a_1 x_i + e_i \quad (i = 1,2,\cdots,n)$$

  - 変数変換により線形関係が得られることがある
    - 変数変換した線形回帰モデル

$$x_i \Rightarrow X_i = \log x_i$$

$$y_i = a_0 + a_1 X_i + e_i \quad (i = 1,2,\cdots,n)$$

  - 相関係数で評価

線形重回帰

- 1つの目的変数yに対する複数(p個)の説明変数$x_1, x_2, \cdots, x_p$
  - 線形重回帰モデル

$$y_i = a_0 + a_1 x_{1i} + a_2 x_{2i} + \cdots + a_p x_{pi} + e_i \qquad (i = 1, 2, \cdots, n)$$

	目的変数:y	説明変数:x	予測誤差
1	y_1	$x_{11}, x_{21}, \cdots, x_{p1}$	$e_1 = y_1 - (a_0 + a_1 x_{11} + a_2 x_{21} + \cdots + a_p x_{p1})$
2	y_2	$x_{12}, x_{22}, \cdots, x_{p2}$	$e_2 = y_2 - (a_0 + a_1 x_{12} + a_2 x_{22} + \cdots + a_p x_{p2})$
…			
i	y_i	$x_{1i}, x_{2i}, \cdots, x_{pi}$	$e_i = y_i - (a_0 + a_1 x_{1i} + a_2 x_{2i} + \cdots + a_p x_{pi})$
…			
n	y_n	$x_{1n}, x_{2n}, \cdots, x_{pn}$	$e_n = y_n - (a_0 + a_1 x_{1n} + a_2 x_{2n} + \cdots + a_p x_{pn})$

線形重回帰

- 単回帰　(x,y)平面
- 重回帰　(x_1,x_2,・・・,x_p,y) (p+1)次元空間
  - P次元超平面

$$y = a_0 + a_1 x_1 + a_2 x_2 + \cdots + a_p x_p$$

  - 説明変数x_{1i},x_{2i},・・・,x_{pi}から目的変数y_iを予測
    - 予測誤差

$$e_i = y_i - \left(a_0 + a_1 x_{1i} + a_2 x_{2i} + \cdots + a_p x_{pi}\right)$$

    - 予測誤差の平方和

$$\sum_{i=1}^{n} e_i^{\ 2} = \sum_{i=1}^{n} \left\{ y_i - \left(a_0 + a_1 x_{1i} + a_2 x_{2i} + \cdots + a_p x_{pi}\right)\right\}^2$$

線形重回帰

- 予測誤差の平方和の最小化

$$F\left(a_0, a_1, \cdots, a_p\right) = \sum_{i=1}^{n} {e_i}^2$$

$$= \sum_{i=1}^{n} \left\{y_i - \left(a_0 + a_1 x_{1i} + a_2 x_{2i} + \cdots + a_p x_{pi}\right)\right\}^2$$

$$\begin{cases} \dfrac{\partial}{\partial a_0} F\left(a_0, a_1, \cdots, a_p\right) = 0 \\ \dfrac{\partial}{\partial a_1} F\left(a_0, a_1, \cdots, a_p\right) = 0 \\ \qquad \vdots \\ \dfrac{\partial}{\partial a_p} F\left(a_0, a_1, \cdots, a_p\right) = 0 \end{cases}$$

線形重回帰

$$\frac{\partial}{\partial a_0} F(a_0, a_1, \cdots, a_p) = \frac{\partial}{\partial a_0} \sum_{i=1}^{n} \{y_i - (a_0 + a_1 x_{1i} + a_2 x_{2i} + \cdots + a_p x_{pi})\}^2$$

$$= -2 \sum_{i=1}^{n} \{y_i - (a_0 + a_1 x_{1i} + a_2 x_{2i} + \cdots + a_p x_{pi})\} = 0$$

$$\frac{\partial}{\partial a_1} F(a_0, a_1, \cdots, a_p) = \frac{\partial}{\partial a_1} \sum_{i=1}^{n} \{y_i - (a_0 + a_1 x_{1i} + a_2 x_{2i} + \cdots + a_p x_{pi})\}^2$$

$$= -2 \sum_{i=1}^{n} x_{1i} \{y_i - (a_0 + a_1 x_{1i} + a_2 x_{2i} + \cdots + a_p x_{pi})\} = 0$$

$$\frac{\partial}{\partial a_p} F(a_0, a_1, \cdots, a_p) = \frac{\partial}{\partial a_p} \sum_{i=1}^{n} \{y_i - (a_0 + a_1 x_{1i} + a_2 x_{2i} + \cdots + a_p x_{pi})$$

$$= -2 \sum_{i=1}^{n} x_{pi} \{y_i - (a_0 + a_1 x_{1i} + a_2 x_{2i} + \cdots + a_p x_{pi})\} = 0$$

線形重回帰

$$\begin{cases} \sum_{i=1}^{n} \left\{ y_i - \left(a_0 + a_1 x_{1i} + a_2 x_{2i} + \cdots + a_p x_{pi} \right) \right\} = 0 \\ \sum_{i=1}^{n} x_{1i} \left\{ y_i - \left(a_0 + a_1 x_{1i} + a_2 x_{2i} + \cdots + a_p x_{pi} \right) \right\} = 0 \\ \vdots \\ \sum_{i=1}^{n} x_{pi} \left\{ y_i - \left(a_0 + a_1 x_{1i} + a_2 x_{2i} + \cdots + a_p x_{pi} \right) \right\} = 0 \end{cases}$$

線形重回帰

- 正規方程式:残差二乗和を最小にする推定値を与える方程式→$a_0, a_1, \cdots, a_p$の連立方程式

$$
\begin{cases}
a_0 n + a_1 \sum_{i=1}^{n} x_{1i} + a_2 \sum_{i=1}^{n} x_{2i} + \cdots + a_p \sum_{i=1}^{n} x_{pi} = \sum_{i=1}^{n} y_i \\
a_0 \sum_{i=1}^{n} x_{1i} + a_1 \sum_{i=1}^{n} x_{1i}{}^2 + a_2 \sum_{i=1}^{n} x_{1i} x_{2i} + \cdots + a_p \sum_{i=1}^{n} x_{1i} x_{pi} = \sum_{i=1}^{n} x_{1i} y_i \\
\vdots \\
a_0 \sum_{i=1}^{n} x_{pi} + a_1 \sum_{i=1}^{n} x_{pi} x_{1i} + a_2 \sum_{i=1}^{n} x_{pi} x_{2i} + \cdots + a_p \sum_{i=1}^{n} x_{pi} x_{pi} = \sum_{i=1}^{n} x_{pi} y_i
\end{cases}
$$

線形重回帰

$$
\begin{bmatrix}
n & \sum_{i=1}^{n} x_{1i} & \sum_{i=1}^{n} x_{2i} & \cdots & \sum_{i=1}^{n} x_{pi} \\
\sum_{i=1}^{n} x_{1i} & \sum_{i=1}^{n} x_{1i}{}^{2} & \sum_{i=1}^{n} x_{1i} x_{2i} & & \sum_{i=1}^{n} x_{1i} x_{pi} \\
\vdots & & & & \vdots \\
\sum_{i=1}^{n} x_{pi} & \sum_{i=1}^{n} x_{pi} x_{1i} & \sum_{i=1}^{n} x_{pi} x_{2i} & & \sum_{i=1}^{n} x_{pi} x_{pi}
\end{bmatrix}
\begin{bmatrix} a_0 \\ a_1 \\ \vdots \\ a_p \end{bmatrix}
=
\begin{bmatrix} \sum_{i=1}^{n} y_i \\ \sum_{i=1}^{n} x_{1i} y_i \\ \vdots \\ \sum_{i=1}^{n} x_{pi} y_i \end{bmatrix}
$$

線形重回帰

$$
\begin{bmatrix} a_0 \\ a_1 \\ \vdots \\ a_p \end{bmatrix} =
\begin{bmatrix}
n & \sum_{i=1}^{n} x_{1i} & \sum_{i=1}^{n} x_{2i} & \cdots & \sum_{i=1}^{n} x_{pi} \\
\sum_{i=1}^{n} x_{1i} & \sum_{i=1}^{n} x_{1i}{}^2 & \sum_{i=1}^{n} x_{1i} x_{2i} & & \sum_{i=1}^{n} x_{1i} x_{pi} \\
\vdots & & & & \vdots \\
\sum_{i=1}^{n} x_{pi} & \sum_{i=1}^{n} x_{pi} x_{1i} & \sum_{i=1}^{n} x_{pi} x_{2i} & & \sum_{i=1}^{n} x_{pi} x_{pi}
\end{bmatrix}^{-1}
\begin{bmatrix} \sum_{i=1}^{n} y_i \\ \sum_{i=1}^{n} x_{1i} y_i \\ \vdots \\ \sum_{i=1}^{n} x_{pi} y_i \end{bmatrix}
$$

$$
a_0, a_1, \cdots, a_p \Rightarrow \hat{a}_0, \hat{a}_1, \cdots, \hat{a}_p
$$

分散共分散で表すことを考える

線形重回帰

- $x_1,\cdots,x_p$の分散共分散行列

$$V = \begin{bmatrix} s_{11} & s_{12} & \cdots & s_{1l} & \cdots & s_{1p} \\ s_{21} & s_{22} & & & & s_{2p} \\ \vdots & & & & & \vdots \\ s_{j1} & s_{j2} & \cdots & s_{jl} & \cdots & s_{jp} \\ \vdots & & & & & \vdots \\ s_{p1} & s_{p2} & \cdots & s_{pl} & \cdots & s_{pp} \end{bmatrix}$$

$$s_{jl} = \frac{1}{n}\sum_{i=1}^{n}\left(x_{ji} - \bar{x}_j\right)\left(x_{li} - \bar{x}_l\right)$$

$$j,l = 1,2,\cdots,p$$

- yと$x_1,\cdots,x_p$の共分散

$$s_{yj} = \frac{1}{n}\sum_{i=1}^{n}\left(y_i - \bar{y}\right)\left(x_{ji} - \bar{x}_j\right) \quad j = 1,2,\cdots,p$$

線形重回帰
分散共分散行列であらわす

- 正規方程式の一行目

$$\hat{a}_0 n + \hat{a}_1 \sum_{i=1}^{n} x_{1i} + \hat{a}_2 \sum_{i=1}^{n} x_{2i} + \cdots + \hat{a}_p \sum_{i=1}^{n} x_{pi} = \sum_{i=1}^{n} y_i$$

$$\hat{a}_0 = \frac{1}{n} \sum_{i=1}^{n} y_i - \frac{1}{n} \left(\hat{a}_1 \sum_{i=1}^{n} x_{1i} + \hat{a}_2 \sum_{i=1}^{n} x_{2i} + \cdots + \hat{a}_p \sum_{i=1}^{n} x_{pi} \right)$$

$$= \bar{y} - \left(\hat{a}_1 \bar{x}_1 + \hat{a}_2 \bar{x}_2 + \cdots + \hat{a}_p \bar{x}_p \right)$$

$$\hat{a}_0 = \bar{y} - \left(\hat{a}_1 \bar{x}_1 + \hat{a}_2 \bar{x}_2 + \cdots + \hat{a}_p \bar{x}_p \right)$$

  - 正規方程式のa_0に代入

線形重回帰

$$
\begin{cases}
\displaystyle\sum_{i=1}^{n}\left\{y_i-\left(\left[\overline{y}-\left(\hat{a}_1\overline{x}_1+\hat{a}_2\overline{x}_2+\cdots+\hat{a}_p\overline{x}_p\right)\right]+\hat{a}_1x_{1i}+\hat{a}_2x_{2i}+\cdots+\hat{a}_px_{pi}\right)\right\}=0 \\
\displaystyle\sum_{i=1}^{n}x_{1i}\left\{y_i-\left(\left[\overline{y}-\left(\hat{a}_1\overline{x}_1+\hat{a}_2\overline{x}_2+\cdots+\hat{a}_p\overline{x}_p\right)\right]+\hat{a}_1x_{1i}+\hat{a}_2x_{2i}+\cdots+\hat{a}_px_{pi}\right)\right\}=0 \\
\vdots \\
\displaystyle\sum_{i=1}^{n}x_{ji}\left\{y_i-\left(\left[\overline{y}-\left(\hat{a}_1\overline{x}_1+\hat{a}_2\overline{x}_2+\cdots+\hat{a}_p\overline{x}_p\right)\right]+\hat{a}_1x_{1i}+\hat{a}_2x_{2i}+\cdots+\hat{a}_px_{pi}\right)\right\}=0 \\
\vdots \\
\displaystyle\sum_{i=1}^{n}x_{pi}\left\{y_i-\left(\left[\overline{y}-\left(\hat{a}_1\overline{x}_1+\hat{a}_2\overline{x}_2+\cdots+\hat{a}_p\overline{x}_p\right)\right]+\hat{a}_1x_{1i}+\hat{a}_2x_{2i}+\cdots+\hat{a}_px_{pi}\right)\right\}=0
\end{cases}
$$

線形重回帰

$$
\begin{cases}
\displaystyle\sum_{i=1}^{n} \underline{\left\{(y_i - \bar{y}) - \hat{a}_1(x_{1i} - \bar{x}_1) - \hat{a}_2(x_{2i} - \bar{x}_2) - \cdots - \hat{a}_p(x_{pi} - \bar{x}_p)\right\}} = 0 \\
\displaystyle\sum_{i=1}^{n} x_{1i} \underline{\left\{(y_i - \bar{y}) - \hat{a}_1(x_{1i} - \bar{x}_1) - \hat{a}_2(x_{2i} - \bar{x}_2) - \cdots - \hat{a}_p(x_{pi} - \bar{x}_p)\right\}} = 0 \\
\vdots \\
\displaystyle\sum_{i=1}^{n} x_{ji} \underline{\left\{(y_i - \bar{y}) - \hat{a}_1(x_{1i} - \bar{x}_1) - \hat{a}_2(x_{2i} - \bar{x}_2) - \cdots - \hat{a}_p(x_{pi} - \bar{x}_p)\right\}} = 0 \\
\vdots \\
\displaystyle\sum_{i=1}^{n} x_{pi} \underline{\left\{(y_i - \bar{y}) - \hat{a}_1(x_{1i} - \bar{x}_1) - \hat{a}_2(x_{2i} - \bar{x}_2) - \cdots - \hat{a}_p(x_{pi} - \bar{x}_p)\right\}} = 0
\end{cases}
$$

各行j=1~pから，j=0行×$\bar{x}_j$を引く　　　　共通

線形重回帰

$$\begin{cases} \sum_{i=1}^{n}\left(x_{1i}-\bar{x}_1\right)\left\{\left(y_i-\bar{y}\right)-\hat{a}_1\left(x_{1i}-\bar{x}_1\right)-\hat{a}_2\left(x_{2i}-\bar{x}_2\right)-\cdots-\hat{a}_p\left(x_{pi}-\bar{x}_p\right)\right\}=0 \\ \vdots \\ \sum_{i=1}^{n}\left(x_{ji}-\bar{x}_j\right)\left\{\left(y_i-\bar{y}\right)-\hat{a}_1\left(x_{1i}-\bar{x}_1\right)-\hat{a}_2\left(x_{2i}-\bar{x}_2\right)-\cdots-\hat{a}_p\left(x_{pi}-\bar{x}_p\right)\right\}=0 \\ \vdots \\ \sum_{i=1}^{n}\left(x_{pi}-\bar{x}_p\right)\left\{\left(y_i-\bar{y}\right)-\hat{a}_1\left(x_{1i}-\bar{x}_1\right)-\hat{a}_2\left(x_{2i}-\bar{x}_2\right)-\cdots-\hat{a}_p\left(x_{pi}-\bar{x}_p\right)\right\}=0 \end{cases}$$

各行の両辺をnで割ると

線形重回帰

$$
\begin{cases}
s_{y1} - \hat{a}_1 s_{11} - \hat{a}_2 s_{12} - \cdots - \hat{a}_p s_{1p} = 0 \\
\vdots \\
s_{yj} - \hat{a}_1 s_{j1} - \hat{a}_2 s_{j2} - \cdots - \hat{a}_p s_{jp} = 0 \\
\vdots \\
s_{yp} - \hat{a}_1 s_{p1} - \hat{a}_2 s_{p2} - \cdots - \hat{a}_p s_{pp} = 0
\end{cases}
$$

$$
\begin{cases}
\hat{a}_1 s_{11} + \hat{a}_2 s_{12} + \cdots + \hat{a}_p s_{1p} = s_{y1} \\
\vdots \\
\hat{a}_1 s_{j1} + \hat{a}_2 s_{j2} + \cdots + \hat{a}_p s_{jp} = s_{yj} \\
\vdots \\
\hat{a}_1 s_{p1} + \hat{a}_2 s_{p2} + \cdots + \hat{a}_p s_{pp} = s_{yp}
\end{cases}
$$

線形重回帰

$$\begin{bmatrix} s_{11} & s_{12} & \cdots & s_{1p} \\ \vdots & & & \\ s_{j1} & s_{j2} & & s_{jp} \\ \vdots & & & \\ s_{p1} & s_{p2} & \cdots & s_{pp} \end{bmatrix} \begin{bmatrix} \hat{a}_1 \\ \hat{a}_2 \\ \vdots \\ \hat{a}_p \end{bmatrix} = \begin{bmatrix} s_{y1} \\ \vdots \\ s_{yj} \\ \vdots \\ s_{yp} \end{bmatrix}$$

分散共分散行列V　　　　yのxに対する共分散

別途　$\hat{a}_0 = \bar{y} - \left(\hat{a}_1\bar{x}_1 + \hat{a}_2\bar{x}_2 + \cdots + \hat{a}_p\bar{x}_p\right)$　で　$\hat{a}_0$を求めればよい

目的変数yの，説明変数x_1,x_2,・・・,x_pに対する線形重回帰式

$$y = \hat{a}_0 + \hat{a}_1 x_1 + \hat{a}_2 x_2 + \cdots + \hat{a}_p x_p$$

重回帰係数　$\hat{a}_j \quad j = 1, 2, \cdots, p$