応用システム工学
第五回　線形カルマンフィルタ

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一括処理と逐次処理

- 測定値:$z(i)$, $i = 1,2,\cdots,k$
  - 平均値の推定値:$\widehat{m}(k)$
    - $\widehat{m}(k) = \frac{z(1)+z(2)+\cdots+z(k)}{k}$
  - 一括処理
    - $\widehat{m}(k+1) = \frac{z(1)+z(2)+\cdots+z(k+1)}{k+1}$
  - 逐次処理(計算機のオンライン処理向き)
    - $n = 1$:$\widehat{m}(1) = z(1)$
    - $n = 2$:$\widehat{m}(2) = \frac{1}{2}\widehat{m}(1) + \frac{1}{2}z(2)$
    - $n = 3$:$\widehat{m}(3) = \frac{2}{3}\widehat{m}(2) + \frac{1}{3}z(3)$

$$n = k:\widehat{m}(k) = \frac{k-1}{k}\widehat{m}(k-1) + \frac{1}{k}z(k)$$

事前推定値と事後推定値

- 現時刻kの1刻み前において状態推定値$\hat{\boldsymbol{x}}(k-1)$が得られている
- 時刻kの観測値$y(k)$を用いて状態推定値$\hat{\boldsymbol{x}}(k)$を求める
- 事前推定値:$\hat{\boldsymbol{x}}^{-}(k)$ $\left(=\hat{\boldsymbol{x}}(k|k-1)\right)$
  - 時刻$k-1$までのデータによる，時刻kにおける$\boldsymbol{x}$の予測推定値（予測ステップ）
- 事後推定値:$\hat{\boldsymbol{x}}(k)$　$\left(=\hat{\boldsymbol{x}}(k|k)\right)$
  - 時刻kまでのデータによる，時刻kにおける$\boldsymbol{x}$のフィルタリング推定値（フィルタリングステップ）

線形予測器

- 線形予測器:事前推定値$\hat{\boldsymbol{x}}^{-}(k)$と観測値$y(k)$の線形モデル
  - $\hat{\boldsymbol{x}}(k) \ = \boldsymbol{G}(k)\hat{\boldsymbol{x}}^{-}(k) + \boldsymbol{g}(k)y(k)$
    - $\boldsymbol{G}(k)$:$n \times n$行列， $\boldsymbol{g}(k)$:n次ベクトル
    - 事前推定値$\hat{\boldsymbol{x}}^{-}$を最新の観測値$y(k)$を用いて修正
    - $\boldsymbol{G}(k)$，$\boldsymbol{g}(k)$ を求めることが必要
- 直交性の原理
  - 観測値$y(i)$は，事後推定誤差$\tilde{\boldsymbol{x}}(k)$と直交する
    - $\tilde{\boldsymbol{x}}(k) = \boldsymbol{x}(k) - \hat{\boldsymbol{x}}(k)$
    - $\mathrm{E}[\tilde{\boldsymbol{x}}(k)y(i)] = 0 \quad\quad i = 1,2,\cdots,k$

観測値と事後推定誤差の直交性

- $\mathrm{E}[\widetilde{\boldsymbol{x}}(k)y(i)] = \mathrm{E}[\{\boldsymbol{x}(k) - \widehat{\boldsymbol{x}}(k)\}y(i)]$
  $= \mathrm{E}[\{\boldsymbol{x}(k) - \langle \boldsymbol{G}(k)\widehat{\boldsymbol{x}}^{-}(k) + \boldsymbol{g}(k)y(k)\rangle\}y(i)]$
  $= \mathrm{E}[\{\boldsymbol{x}(k) - \boldsymbol{G}(k)\widehat{\boldsymbol{x}}^{-}(k) - \boldsymbol{g}(k)y(k)\}y(i)]$
  $= \mathrm{E}[\{\boldsymbol{x}(k) - \boldsymbol{G}(k)\widehat{\boldsymbol{x}}^{-}(k)$
  $- \boldsymbol{g}(k)\langle \boldsymbol{c}^T\boldsymbol{x}(k) + w(k)\rangle\}y(i)]$
  - 観測雑音$w(k)$と過去の観測値$y(i)$, $i = 1,2,\cdots,k-1$は無相関
    - $\mathrm{E}[w(k)y(i)] = 0 \qquad i = 1,2,\cdots,k-1$
- $\mathrm{E}[\widetilde{\boldsymbol{x}}(k)y(i)] = \mathrm{E}[\{\boldsymbol{x}(k) - \boldsymbol{G}(k)\widehat{\boldsymbol{x}}^{-}(k) - \boldsymbol{g}(k)\boldsymbol{c}^T\boldsymbol{x}(k)\}y(i)]$

観測値と事後推定誤差の直交性

- $\mathrm{E}[\widetilde{\boldsymbol{x}}(k)y(i)] =$
  $\mathrm{E}\left[\begin{Bmatrix}\boldsymbol{x}(k) - \boldsymbol{G}(k)\widehat{\boldsymbol{x}}^{-}(k) - \boldsymbol{G}(k)\boldsymbol{x}(k) \\ +\boldsymbol{G}(k)\boldsymbol{x}(k) - \boldsymbol{g}(k)\boldsymbol{c}^T\boldsymbol{x}(k)\end{Bmatrix} y(i)\right]$
  $= \mathrm{E}\begin{bmatrix}\{I - \boldsymbol{G}(k) - \boldsymbol{g}(k)\boldsymbol{c}^T\}\boldsymbol{x}(k)y(i) \\ +\boldsymbol{G}(k)\{\boldsymbol{x}(k) - \widehat{\boldsymbol{x}}^{-}(k)\}y(i)\end{bmatrix}$
  $= \mathrm{E}[\{I - \boldsymbol{G}(k) - \boldsymbol{g}(k)\boldsymbol{c}^T\}\boldsymbol{x}(k)y(i)$
  $+ \boldsymbol{G}(k)\widetilde{\boldsymbol{x}}^{-}(k)y(i)]$
  - 事前推定誤差$\widetilde{\boldsymbol{x}}^{-}(k)$は，観測値$y(i)$と直交
    - $\widetilde{\boldsymbol{x}}^{-}(k) = \boldsymbol{x}(k) - \widehat{\boldsymbol{x}}^{-}(k)$
    - $\mathrm{E}[\widetilde{\boldsymbol{x}}^{-}(k)y(i)] = 0$

観測値と事後推定誤差の直交性

- $\mathrm{E}[\widetilde{\boldsymbol{x}}(k)y(i)] = \mathrm{E}[\{I - \boldsymbol{G}(k) - \boldsymbol{g}(k)\boldsymbol{c}^T\}\boldsymbol{x}(k)y(i)]$
  $= \{I - \boldsymbol{G}(k) - \boldsymbol{g}(k)\boldsymbol{c}^T\}\mathrm{E}[\boldsymbol{x}(k)y(i)]$
  - ここで$\mathrm{E}[\boldsymbol{x}(k)y(i)] \neq 0$なので，$\mathrm{E}[\widetilde{\boldsymbol{x}}(k)y(i)] = 0$とするには
    - $I - \boldsymbol{G}(k) - \boldsymbol{g}(k)\boldsymbol{c}^T = 0$
    - $\boldsymbol{G}(k) = I - \boldsymbol{g}(k)\boldsymbol{c}^T$

観測値と事後推定誤差の直交性

- 線形予測器の式
  - $\hat{\boldsymbol{x}}(k) = \boldsymbol{G}(k)\hat{\boldsymbol{x}}^{-}(k) + \boldsymbol{g}(k)y(k)$
    $= \{I - \boldsymbol{g}(k)\boldsymbol{c}^T\}\hat{\boldsymbol{x}}^{-}(k) + \boldsymbol{g}(k)y(k)$
    $= \boldsymbol{g}(k)\{y(k) - \boldsymbol{c}^T\hat{\boldsymbol{x}}^{-}(k)\} + \hat{\boldsymbol{x}}^{-}(k)$
    - 時刻$k-1$までの出力$y(i)$による，時刻kの事前推定値（1段先予測値）:$\hat{y}^{-}(k)$
      - $\hat{y}^{-}(k) = \boldsymbol{c}^T\hat{\boldsymbol{x}}^{-}(k)$
  - $\hat{\boldsymbol{x}}(k) = \boldsymbol{g}(k)\{y(k) - \hat{y}^{-}(k)\} + \hat{\boldsymbol{x}}^{-}(k)$

観測値と事後推定誤差の直交性

- 出力予測誤差（イノベーション過程）
  - $\tilde{y}(k) = y(k) - \hat{y}^{-}(k)$
    $= y(k) - \boldsymbol{c}^T\hat{\boldsymbol{x}}^{-}(k)$
    $= \boldsymbol{c}^T\boldsymbol{x}(k) + w(k) - \boldsymbol{c}^T\hat{\boldsymbol{x}}^{-}(k)$
    $= \boldsymbol{c}^T\{\boldsymbol{x}(k) - \hat{\boldsymbol{x}}^{-}(k)\} + w(k)$
    $= \boldsymbol{c}^T\tilde{\boldsymbol{x}}^{-}(k) + w(k)$

- 線形予測器の式
  - $\hat{\boldsymbol{x}}(k) = \boldsymbol{g}(k)\tilde{y}(k) + \hat{\boldsymbol{x}}^{-}(k)$
    - $\boldsymbol{g}(k)$:カルマンゲイン
    - 事後推定値$\hat{\boldsymbol{x}}(k)$は，事前推定値$\hat{\boldsymbol{x}}^{-}(k)$と出力予測誤差とカルマンゲインの積の和としてあらわされる
      - カルマンゲインを求める

カルマンゲインの決定

- 観測値$y(i)$と事後推定誤差$\tilde{\boldsymbol{x}}(k)$は直交する
  - $\mathrm{E}[\tilde{\boldsymbol{x}}(k)y(i)] = 0 \qquad i = 1,2,\cdots,k$
    - これが成り立つ条件を求める
  - 出力の事前推定値$\hat{y}^{-}(k)$は，$k-1$までの出力$\hat{y}(i)$の線形和で表される
    - 各$\mathrm{E}[\tilde{\boldsymbol{x}}(k)y(i)] = 0$より，$\mathrm{E}[\tilde{\boldsymbol{x}}(k)\hat{y}^{-}(k)] = 0$となる
  - $\tilde{\boldsymbol{x}}(k) = \boldsymbol{x}(k) - \hat{\boldsymbol{x}}(k)$
  - $\tilde{y}(k) = y(k) - \hat{y}^{-}(k)$より
    - $\hat{y}^{-}(k) = y(k) - \tilde{y}(k)$
  - $\mathrm{E}[\tilde{\boldsymbol{x}}(k)\hat{y}^{-}(k)] = \mathrm{E}[\{\boldsymbol{x}(k) - \hat{\boldsymbol{x}}(k)\}\{y(k) - \tilde{y}(k)\}] = 0$が成り立てばよい

カルマンゲインの決定

- 事後状態誤差ベクトル:$\tilde{\boldsymbol{x}}(k)$
  - $\tilde{\boldsymbol{x}}(k) = \boldsymbol{x}(k) - \hat{\boldsymbol{x}}(k)$

$$
\begin{aligned}
&= \boldsymbol{x}(k) - [\boldsymbol{g}(k)\{y(k) - \boldsymbol{c}^T\hat{\boldsymbol{x}}^-(k)\} + \hat{\boldsymbol{x}}^-(k)] \\
&= \boldsymbol{x}(k) - \hat{\boldsymbol{x}}^-(k) - \boldsymbol{g}(k)\{y(k) - \boldsymbol{c}^T\hat{\boldsymbol{x}}^-(k)\} \\
&= \tilde{\boldsymbol{x}}^-(k) - \boldsymbol{g}(k)\{\boldsymbol{c}^T\boldsymbol{x}(k) + w(k) - \boldsymbol{c}^T\hat{\boldsymbol{x}}^-(k)\} \\
&= \tilde{\boldsymbol{x}}^-(k) - \boldsymbol{g}(k)\{\boldsymbol{c}^T[\boldsymbol{x}(k) - \hat{\boldsymbol{x}}^-(k)] + w(k)\} \\
&= \tilde{\boldsymbol{x}}^-(k) - \boldsymbol{g}(k)\{\boldsymbol{c}^T\tilde{\boldsymbol{x}}^-(k) + w(k)\} \\
&= [\boldsymbol{I} - \boldsymbol{g}(k)\boldsymbol{c}^T]\tilde{\boldsymbol{x}}^-(k) - \boldsymbol{g}(k)w(k)
\end{aligned}
$$

カルマンゲインの決定

- $\mathrm{E}[\widetilde{\boldsymbol{x}}(k)\hat{y}^{-}(k)] = \mathrm{E}[\widetilde{\boldsymbol{x}}(k)\{y(k) - \tilde{y}(k)\}]$
  $= \mathrm{E}[\widetilde{\boldsymbol{x}}(k)y(k)] - \mathrm{E}[\widetilde{\boldsymbol{x}}(k)\tilde{y}(k)] = 0$
  - 事後状態誤差ベクトルと観測値は無相関
    - $\mathrm{E}[\widetilde{\boldsymbol{x}}(k)y(k)] = 0$
  - $\mathrm{E}[\widetilde{\boldsymbol{x}}(k)\tilde{y}(k)] = 0$が成り立てばよい
- $\widetilde{\boldsymbol{x}}(k) = [\boldsymbol{I} - \boldsymbol{g}(k)\boldsymbol{c}^T]\widetilde{\boldsymbol{x}}^{-}(k) - \boldsymbol{g}(k)w(k)$
- $\tilde{y}(k) = \boldsymbol{c}^T\widetilde{\boldsymbol{x}}^{-}(k) + w(k)$
- $\mathrm{E}[\widetilde{\boldsymbol{x}}(k)\tilde{y}(k)]$
  $= \mathrm{E}\left[\{[\boldsymbol{I} - \boldsymbol{g}(k)\boldsymbol{c}^T]\widetilde{\boldsymbol{x}}^{-}(k) - \boldsymbol{g}(k)w(k)\}\{\boldsymbol{c}^T\widetilde{\boldsymbol{x}}^{-}(k) + w(k)\}\right]$

カルマンゲインの決定

- $\mathrm{E}[\widetilde{\boldsymbol{x}}(k)\tilde{y}(k)] = \mathrm{E}\Big[[\boldsymbol{I} - \boldsymbol{g}(k)\boldsymbol{c}^T]\widetilde{\boldsymbol{x}}^-(k)\boldsymbol{c}^T\widetilde{\boldsymbol{x}}^-(k)\Big] + \mathrm{E}\Big[[\boldsymbol{I} - \boldsymbol{g}(k)\boldsymbol{c}^T]\widetilde{\boldsymbol{x}}^-(k)w(k)\Big] - \mathrm{E}[\boldsymbol{g}(k)w(k)w(k)]$
  $= \{\boldsymbol{I} - \boldsymbol{g}(k)\boldsymbol{c}^T\}\mathrm{E}[\widetilde{\boldsymbol{x}}^-(k)\boldsymbol{c}^T\widetilde{\boldsymbol{x}}^-(k)]$
  $+ \{\boldsymbol{I} - \boldsymbol{g}(k)\boldsymbol{c}^T\}\mathrm{E}[\widetilde{\boldsymbol{x}}^-(k)w(k)]$
  $- \boldsymbol{g}(k)\mathrm{E}[w(k)\boldsymbol{c}^T\widetilde{\boldsymbol{x}}(k)] - \boldsymbol{g}(k)\mathrm{E}[w(k)^2]$

カルマンゲインの決定

- $\boldsymbol{c}^T\widetilde{\boldsymbol{x}}(k) = \widetilde{\boldsymbol{x}}(k)^T\boldsymbol{c}$
- 観測雑音$w(k)$と状態$\boldsymbol{x}(k)$は無相関
  - 事前状態推定誤差$\widetilde{\boldsymbol{x}}^-(k)$とも無相関
    - $\mathrm{E}[\widetilde{\boldsymbol{x}}^-(k)w(k)] = \mathrm{E}[w(k)\widetilde{\boldsymbol{x}}^-(k)] = 0$
  - $\mathrm{E}[w(k)^2] = {\sigma_w}^2$
- $\mathrm{E}[\widetilde{\boldsymbol{x}}(k)\tilde{y}(k)] = \{\boldsymbol{I} - \boldsymbol{g}(k)\boldsymbol{c}^T\}\mathrm{E}[\widetilde{\boldsymbol{x}}^-(k)\widetilde{\boldsymbol{x}}^-(k)^T]\boldsymbol{c} + \{\boldsymbol{I} - \boldsymbol{g}(k)\boldsymbol{c}^T\}\mathrm{E}[\widetilde{\boldsymbol{x}}^-(k)w(k)] - \boldsymbol{g}(k)\mathrm{E}[w(k)\widetilde{\boldsymbol{x}}^-(k)^T]\boldsymbol{c} - \boldsymbol{g}(k)\mathrm{E}[w(k)^2]$
  $= \{\boldsymbol{I} - \boldsymbol{g}(k)\boldsymbol{c}^T\}\mathrm{E}[\widetilde{\boldsymbol{x}}^-(k)\widetilde{\boldsymbol{x}}^-(k)^T]\boldsymbol{c} - \boldsymbol{g}(k){\sigma_w}^2$

カルマンゲインの決定

- 事前共分散行列
  - $\boldsymbol{P}^{-}(k) = \mathrm{E}[\widetilde{\boldsymbol{x}}^{-}(k)\widetilde{\boldsymbol{x}}^{-}(k)^T]$
    $= \mathrm{E}[\{\boldsymbol{x}(k) - \widehat{\boldsymbol{x}}^{-}(k)\}\{\boldsymbol{x}(k) - \widehat{\boldsymbol{x}}^{-}(k)\}^T]$
- $\mathrm{E}[\widetilde{\boldsymbol{x}}(k)\tilde{y}(k)] = \{\boldsymbol{I} - \boldsymbol{g}(k)\boldsymbol{c}^T\}\boldsymbol{P}^{-}(k)\boldsymbol{c} - \boldsymbol{g}(k){\sigma_w}^2$
  - $\mathrm{E}[\widetilde{\boldsymbol{x}}(k)\tilde{y}(k)] = 0$
  - $\boldsymbol{P}^{-}(k)\boldsymbol{c} - \boldsymbol{g}(k)\boldsymbol{c}^T\boldsymbol{P}^{-}(k)\boldsymbol{c} - \boldsymbol{g}(k){\sigma_w}^2$
    $= \boldsymbol{P}^{-}(k)\boldsymbol{c} - \boldsymbol{g}(k)\boldsymbol{c}^T\boldsymbol{P}^{-}(k)\boldsymbol{c} - \boldsymbol{g}(k){\sigma_w}^2$
    $= \boldsymbol{P}^{-}(k)\boldsymbol{c} - \boldsymbol{g}(k)\{\boldsymbol{c}^T\boldsymbol{P}^{-}(k)\boldsymbol{c} - {\sigma_w}^2\} = 0$
  - $\boldsymbol{g}(k) = \dfrac{\boldsymbol{P}^{-}(k)\boldsymbol{c}}{\boldsymbol{c}^T\boldsymbol{P}^{-}(k)\boldsymbol{c} - {\sigma_w}^2}$

$\boldsymbol{c}^T\boldsymbol{P}^{-}\boldsymbol{c}(k)$はスカラー

カルマンゲインが求まった

共分散行列の更新

- 時刻$k-1$における事後共分散行列$\boldsymbol{P}(k-1)$を用いて，時刻kにおける事前共分散行列$\boldsymbol{P}^{-}(k)$を求める
  - 状態方程式
    - $\boldsymbol{x}(k) = \boldsymbol{A}\boldsymbol{x}(k-1) + \boldsymbol{b}v(k-1)$
    - 状態の事前推定値$\hat{\boldsymbol{x}}^{-}(k)$を，右辺の期待値として求める
      - $\hat{\boldsymbol{x}}^{-}(k) = \mathrm{E}[\boldsymbol{A}\boldsymbol{x}(k-1) + \boldsymbol{b}v(k-1)]$
        $= \boldsymbol{A}\mathrm{E}[\boldsymbol{x}(k-1)] + \boldsymbol{b}\mathrm{E}[v(k-1)]$
        $= \boldsymbol{A}\mathrm{E}[\boldsymbol{x}(k-1)]$
        $= \boldsymbol{A}\hat{\boldsymbol{x}}(k-1)$

雑音の期待値は0

共分散行列の更新

- 状態の事前推定誤差
  - $\tilde{\boldsymbol{x}}^{-}(k) = \boldsymbol{x}(k) - \hat{\boldsymbol{x}}^{-}(k)$
    $= \{\boldsymbol{A}\boldsymbol{x}(k-1) + \boldsymbol{b}v(k-1)\} - \boldsymbol{A}\hat{\boldsymbol{x}}(k-1)$
    $= \boldsymbol{A}\{\boldsymbol{x}(k-1) - \hat{\boldsymbol{x}}(k-1)\} + \boldsymbol{b}v(k-1)$
    $= \boldsymbol{A}\tilde{\boldsymbol{x}}(k-1) + \boldsymbol{b}v(k-1)$

共分散行列の更新

- 状態の事前共分散行列
  - $\boldsymbol{P}^{-}(k) = \mathrm{E}[\widetilde{\boldsymbol{x}}^{-}(k)\widetilde{\boldsymbol{x}}^{-}(k)^{T}]$
    $= \mathrm{E}[\{\boldsymbol{A}\widetilde{\boldsymbol{x}}(k-1)$
    $+ \boldsymbol{b}v(k-1)\}\{\boldsymbol{A}\widetilde{\boldsymbol{x}}(k-1) + \boldsymbol{b}v(k-1)\}^{T}]$
    $= \mathrm{E}[\{\boldsymbol{A}\widetilde{\boldsymbol{x}}(k-1) + \boldsymbol{b}v(k-1)\}\{\widetilde{\boldsymbol{x}}(k-1)^{\boldsymbol{T}}\boldsymbol{A}^{\boldsymbol{T}}$
    $+ v(k-1)^{\boldsymbol{T}}\boldsymbol{b}^{\boldsymbol{T}}\}]$
    $= \mathrm{E}[\boldsymbol{A}\widetilde{\boldsymbol{x}}(k-1)\widetilde{\boldsymbol{x}}(k-1)^{\boldsymbol{T}}\boldsymbol{A}^{\boldsymbol{T}}]$
    $+ \mathrm{E}[\boldsymbol{A}\widetilde{\boldsymbol{x}}(k-1)v(k-1)^{\boldsymbol{T}}\boldsymbol{b}^{\boldsymbol{T}}]$
    $+ \mathrm{E}[\boldsymbol{b}v(k-1)\widetilde{\boldsymbol{x}}(k-1)^{\boldsymbol{T}}\boldsymbol{A}^{\boldsymbol{T}}]$
    $+ \mathrm{E}[\boldsymbol{b}v(k-1)v(k-1)^{\boldsymbol{T}}\boldsymbol{b}^{\boldsymbol{T}}]$

共分散行列の更新

- 状態の事前共分散行列
  - $\boldsymbol{P}^{-}(k) = \boldsymbol{A}\mathrm{E}[\widetilde{\boldsymbol{x}}(k-1)\widetilde{\boldsymbol{x}}(k-1)^{\boldsymbol{T}}]\boldsymbol{A}^{\boldsymbol{T}} + \boldsymbol{A}\mathrm{E}[\widetilde{\boldsymbol{x}}(k-1)v(k-1)^{\boldsymbol{T}}]\boldsymbol{b}^{\boldsymbol{T}} + \boldsymbol{b}\mathrm{E}[v(k-1)\widetilde{\boldsymbol{x}}(k-1)^{\boldsymbol{T}}]\boldsymbol{A}^{\boldsymbol{T}} + \boldsymbol{b}\mathrm{E}[v(k-1)v(k-1)^{\boldsymbol{T}}]\boldsymbol{b}^{\boldsymbol{T}}$
  - 状態推定誤差$\widetilde{\boldsymbol{x}}(k-1)$とシステム雑音$v(k-1)$は無相関
    - $\mathrm{E}[\widetilde{\boldsymbol{x}}(k-1)v(k-1)^{\boldsymbol{T}}] = \mathrm{E}[v(k-1)\widetilde{\boldsymbol{x}}(k-1)^{\boldsymbol{T}}] = 0$
    - $\boldsymbol{P}^{-}(k) = \boldsymbol{A}\mathrm{E}[\widetilde{\boldsymbol{x}}(k-1)\widetilde{\boldsymbol{x}}(k-1)^{\boldsymbol{T}}]\boldsymbol{A}^{\boldsymbol{T}} + \boldsymbol{b}\mathrm{E}[v(k-1)v(k-1)^{\boldsymbol{T}}]\boldsymbol{b}^{\boldsymbol{T}}$

共分散行列の更新

- 時刻$k-1$における事後誤差共分散行列$\boldsymbol{P}(k-1)$
  - $\boldsymbol{P}(k-1) = \mathrm{E}[\widetilde{\boldsymbol{x}}(k-1)\widetilde{\boldsymbol{x}}(k-1)^{\boldsymbol{T}}]$
- $\mathrm{E}[v(k-1)v(k-1)^{\boldsymbol{T}}] = {\sigma_v}^{\boldsymbol{2}}$
- $\boldsymbol{P}^{-}(k) = \boldsymbol{A}\boldsymbol{P}(k-1)\boldsymbol{A}^{\boldsymbol{T}} + {\sigma_v}^{2}\boldsymbol{b}\boldsymbol{b}^{\boldsymbol{T}}$
- 事前共分散行列$\boldsymbol{P}^{-}(k)$を用いて時刻kの事後共分散行列$\boldsymbol{P}(k)$を求める
  - $\boldsymbol{P}(k) = \mathrm{E}[\widetilde{\boldsymbol{x}}(k)\widetilde{\boldsymbol{x}}(k)^{\boldsymbol{T}}]$
    - $\widetilde{\boldsymbol{x}}(k) = [\boldsymbol{I} - \boldsymbol{g}(k)\boldsymbol{c}^{T}]\widetilde{\boldsymbol{x}}^{-}(k) - \boldsymbol{g}(k)w(k)$より

共分散行列の更新

- $\boldsymbol{P}(k) = \mathrm{E}[\widetilde{\boldsymbol{x}}(k)\widetilde{\boldsymbol{x}}(k)^{\boldsymbol{T}}]$
  $= \mathrm{E}[\{[\boldsymbol{I} - \boldsymbol{g}(k)\boldsymbol{c}^T]\widetilde{\boldsymbol{x}}^-(k) - \boldsymbol{g}(k)w(k)\}$
  $\{[\boldsymbol{I} - \boldsymbol{g}(k)\boldsymbol{c}^T]\widetilde{\boldsymbol{x}}^-(k) - \boldsymbol{g}(k)w(k)\}^{\boldsymbol{T}}]$
  $= \mathrm{E}[\{[\boldsymbol{I} - \boldsymbol{g}(k)\boldsymbol{c}^T]\widetilde{\boldsymbol{x}}^-(k) - \boldsymbol{g}(k)w(k)\}$
  $\{\widetilde{\boldsymbol{x}}^-(k)^T[\boldsymbol{I} - \boldsymbol{g}(k)\boldsymbol{c}^T]^T - w(k)^T\boldsymbol{g}(k)^T\}]$
  $= C_1 + C_2 + C_3 + C_4$
  - $C_1 = \mathrm{E}\big[[\boldsymbol{I} - \boldsymbol{g}(k)\boldsymbol{c}^T]\widetilde{\boldsymbol{x}}^-(k)[\boldsymbol{I} - \boldsymbol{g}(k)\boldsymbol{c}^T]\widetilde{\boldsymbol{x}}^-(k)\big]$
  - $C_2 = -\mathrm{E}\big[[\boldsymbol{I} - \boldsymbol{g}(k)\boldsymbol{c}^T]\widetilde{\boldsymbol{x}}^-(k)w(k)^T\boldsymbol{g}(k)^T\big]$
  - $C_3 = -\mathrm{E}[\boldsymbol{g}(k)w(k)[\boldsymbol{I} - \boldsymbol{g}(k)\boldsymbol{c}^T]\widetilde{\boldsymbol{x}}^-(k)]$
  - $C_4 = \mathrm{E}[\boldsymbol{g}(k)w(k)w(k)^T\boldsymbol{g}(k)^T]$

共分散行列の更新

- $C_1 = \mathrm{E}\left[[\boldsymbol{I} - \boldsymbol{g}(k)\boldsymbol{c}^T]\widetilde{\boldsymbol{x}}^-(k)[\boldsymbol{I} - \boldsymbol{g}(k)\boldsymbol{c}^T]\widetilde{\boldsymbol{x}}^-(k)\right]$
$= \mathrm{E}\left[[\boldsymbol{I} - \boldsymbol{g}(k)\boldsymbol{c}^T]\widetilde{\boldsymbol{x}}^-(k)\widetilde{\boldsymbol{x}}^-(k)^T[\boldsymbol{I} - \boldsymbol{g}(k)\boldsymbol{c}^T]^T\right]$
$= [\boldsymbol{I} - \boldsymbol{g}(k)\boldsymbol{c}^T]\mathrm{E}[\widetilde{\boldsymbol{x}}^-(k)\widetilde{\boldsymbol{x}}^-(k)^T][\boldsymbol{I} - \boldsymbol{g}(k)\boldsymbol{c}^T]^T$

共分散行列の更新

- $C_2 = -\mathrm{E}\left[[\boldsymbol{I} - \boldsymbol{g}(k)\boldsymbol{c}^T]\tilde{\boldsymbol{x}}^-(k)w(k)^T\boldsymbol{g}(k)^T\right]$
  $= -\mathrm{E}\left[[\boldsymbol{I} - \boldsymbol{g}(k)\boldsymbol{c}^T]\tilde{\boldsymbol{x}}^-(k)w(k)^T\boldsymbol{g}(k)^T\right]$
  $= -[\boldsymbol{I} - \boldsymbol{g}(k)\boldsymbol{c}^T]\mathrm{E}[\tilde{\boldsymbol{x}}^-(k)w(k)^T]\boldsymbol{g}(k)^T$
- $C_3 = -\mathrm{E}[\boldsymbol{g}(k)w(k)[\boldsymbol{I} - \boldsymbol{g}(k)\boldsymbol{c}^T]\tilde{\boldsymbol{x}}^-(k)]$
  $= -\mathrm{E}[\boldsymbol{g}(k)w(k)\tilde{\boldsymbol{x}}^-(k)^T[\boldsymbol{I} - \boldsymbol{g}(k)\boldsymbol{c}^T]^T]$
  $= -\boldsymbol{g}(k)\mathrm{E}[w(k)\tilde{\boldsymbol{x}}^-(k)^T][\boldsymbol{I} - \boldsymbol{g}(k)\boldsymbol{c}^T]^T$
- $C_4 = \mathrm{E}[\boldsymbol{g}(k)w(k)w(k)^T\boldsymbol{g}(k)^T]$
  $= \mathrm{E}[\boldsymbol{g}(k)w(k)w(k)^T\boldsymbol{g}(k)^T]$
  $= \boldsymbol{g}(k)\mathrm{E}[w(k)w(k)^T]\boldsymbol{g}(k)^T$

共分散行列の更新

- $\mathrm{E}[\widetilde{\boldsymbol{x}}^{-}(k)w(k)^{T}] = \mathrm{E}[w(k)\widetilde{\boldsymbol{x}}^{-}(k)^{T}] = 0$より
  - $C_2 = -[\boldsymbol{I} - \boldsymbol{g}(k)\boldsymbol{c}^{T}]\mathrm{E}[\widetilde{\boldsymbol{x}}^{-}(k)w(k)^{T}]\boldsymbol{g}(k)^{T} = 0$
  - $C_3 = -\boldsymbol{g}(k)\mathrm{E}[w(k)\widetilde{\boldsymbol{x}}^{-}(k)^{T}][\boldsymbol{I} - \boldsymbol{g}(k)\boldsymbol{c}^{T}]^{T} = 0$
  - $\boldsymbol{P}(k) = C_1 + C_4$
    $= [\boldsymbol{I} - \boldsymbol{g}(k)\boldsymbol{c}^{T}]\mathrm{E}[\widetilde{\boldsymbol{x}}^{-}(k)\widetilde{\boldsymbol{x}}^{-}(k)^{T}][\boldsymbol{I} - \boldsymbol{g}(k)\boldsymbol{c}^{T}]^{T}$
    $+\boldsymbol{g}(k)\mathrm{E}[w(k)w(k)^{T}]\boldsymbol{g}(k)^{T}$
    - $\mathrm{E}[\widetilde{\boldsymbol{x}}^{-}(k)\widetilde{\boldsymbol{x}}^{-}(k)^{T}] = \boldsymbol{P}^{-}(k)$
    - $\mathrm{E}[w(k)w(k)^{T}] = {\boldsymbol{\sigma}_{w}}^{2}$
  - $\boldsymbol{P}(k) = [\boldsymbol{I} - \boldsymbol{g}(k)\boldsymbol{c}^{T}]\boldsymbol{P}^{-}(k)[\boldsymbol{I} - \boldsymbol{g}(k)\boldsymbol{c}^{T}]^{T} + \boldsymbol{g}(k){\boldsymbol{\sigma}_{w}}^{2}\boldsymbol{g}(k)^{T}$

共分散行列の更新

- $\{\boldsymbol{I}-\boldsymbol{g}(k)\boldsymbol{c}^T\}\boldsymbol{P}^-(k)\boldsymbol{c}-\boldsymbol{g}(k){\sigma_w}^2=0$より
$$\{\boldsymbol{I}-\boldsymbol{g}(k)\boldsymbol{c}^T\}\boldsymbol{P}^-(k)\boldsymbol{c}=\boldsymbol{g}(k){\sigma_w}^2$$
- $\boldsymbol{P}(k)=[\boldsymbol{I}-\boldsymbol{g}(k)\boldsymbol{c}^T]\boldsymbol{P}^-(k)-[\boldsymbol{I}-\boldsymbol{g}(k)\boldsymbol{c}^T]\boldsymbol{P}^-(k)\boldsymbol{c}\boldsymbol{g}(k)^T+\boldsymbol{g}(k){\boldsymbol{\sigma}_w}^2\boldsymbol{g}(k)^T(k)$
$$=[\boldsymbol{I}-\boldsymbol{g}(k)\boldsymbol{c}^T]\boldsymbol{P}^-(k)-\boldsymbol{g}(k){\sigma_w}^2\boldsymbol{g}(k)^T+\boldsymbol{g}(k){\boldsymbol{\sigma}_w}^2\boldsymbol{g}(k)^T$$
$$=[\boldsymbol{I}-\boldsymbol{g}(k)\boldsymbol{c}^T]\boldsymbol{P}^-(k)$$
- 上式で共分散行列を更新する